Module Overview	
For Grade Level(s)	Middle School: 8th grade High School: 9th, 10th, and 11th grades
Suggested Time	MESA Period: 1 to 2 weeks of daily 50-60 minute sessions MESA Afterschool: 3 total sessions of 60-90 minutes each MESA Saturday: 1 Saturday for total of 6 hours
Purpose	The purpose of this unit is to better prepare MESA students for the math section of the PSAT and SAT. In this MESA Day contest, students participate in an exam that simulates the math section of the redesigned SAT and measures competency in: • Mastery of linear equations and systems • Problem solving and data analysis • Manipulation of complex equations • Geometric and trigonometric skills most relevant to college and career readiness **The MESA Day Solo Math contest will differ in the time allotted AND will NOT include student-produced response questions.**
Objectives	Students will be able to: • State the mathematics skills measured by the SAT • State the general SAT math test taking rules • Explain the format of the math section of the SAT test • List the mathematics content and types of mathematical questions on the SAT test
Contents	Lesson # 1: Heart of Algebra Lesson # 2: Problem Solving and Data Analysis Lesson # 3: Passport to Advanced Math Lesson # 4: Additional Topics in Math Lesson # 5: Practice on Your Own
Standards Addressed (Common Core)	A key priority of the math common core is to cover fewer topics in greater depth. Also, Common Core states, "The high school standards specify the mathematics that all students should study in order to be college and career ready." The math section of the redesigned SAT draws from fewer topics that evidence shows most contribute to student readiness for college and career training. Students can study these core math areas in depth and have confidence that they will be assessed.

	The Common Core standards are too lengthy to list here. However, below are the Common Core Standards for Mathematical Practice: 1. Make sense of problems and persevere in solving them. 2. Reason abstractly and quantitatively. 3. Construct viable arguments and critique the reasoning of others. 4. Model with mathematics. 5. Use appropriate tools strategically. 6. Attend to precision. 7. Look for and make use of structure. 8. Look for and express regularity in repeated reasoning.
Assessment	Students will be evaluated through the following methods: • Assessment worksheets • Solo Math contest at Pre MESA Day and Regional MESA Day
Additional Resources	College Board SAT Suite of Assessments https://collegereadiness.collegeboard.org/ College Board PSAT 8/9 https://collegereadiness.collegeboard.org/psat-8-9 Sample Math Questions https://collegereadiness.collegeboard.org/sample-questions/math Khan Academy (Free) https://www.khanacademy.org/sat

Note: Information for this guide has been taken from the College Board's website at https://collegereadiness.collegeboard.org/. Information is to be used in an educational, noncommercial setting that is not intended for commercial advantage or private monetary compensation. The College Board and College Board affiliates exclusively own the copyright to the information, data, and all contents provided. ©2015 The College Board

Background / General Information on the redesigned SAT and PSAT 8/9

The redesigned SAT is the anchor of the SAT Suite of Assessments. Tightly aligned with the PSAT/NMSQT, PSAT 10, and PSAT 8/9, it provides a powerful connection to college and career. Students can take the redesigned SAT starting March 2016 and the PSAT 8/9 beginning in fall of 2015.

The redesigned SAT and the PSAT 8/9 include the following components:

- Evidence-Based Reading and Writing
  - Reading Test
  - Writing and Language Test
- Math
- Essay (optional)

Total testing time* is 3 hours (plus 50 minutes for the Essay [optional]).

** Redesigned SAT testing time subject to research*

Redesigned SAT Test Length and Timing Compared to Current/Old SAT

Comparison of Test Length and Timing: Current SAT and Redesigned SAT					
Current/Old SAT			**Redesigned SAT**		
Component	**Time Allotted (min.)**	**Number of Questions/ Tasks**	**Component**	**Time Allotted (min.)**	**Number of Questions/ Tasks**
Critical Reading	70	67	**Reading**	65	52
Writing	60	49	**Writing and Language**	35	44
Essay	25	1	**Essay (optional)**	50	1
Mathematics	70	54	**Math**	80	58
Total	225	171	**Total**	180 (230 with Essay)	154 (155 with Essay)

PSAT 8/9 Test Length and Timing Compared to ReadiStep

Comparison of Test Length and Timing: ReadiStep and PSAT 8/9					
ReadiStep			**PSAT 8/9**		
Component	**Time Allotted (min.)**	**Number of Questions/ Tasks**	**Component**	**Time Allotted (min.)**	**Number of Questions/ Tasks**
Critical Reading	40	45	**Reading**	55	42
Writing	40	50	**Writing and Language**	30	40
Mathematics	40	36	**Math**	60	38
Total	120	131	**Total**	145	120

Note that the time allotted for the MESA Day contest will be 50 minutes for both high school and middle school. Please see Solo Math Rule # 4 for the details.

Math Test

The Math Test focuses on the math that matters most to college and career readiness. To succeed on the Math Test, students will need to demonstrate mathematical practices, such as problem solving and using appropriate tools strategically.

Every assessment in the redesigned SAT Suite of Assessments will include a Math Test that covers the content, knowledge, and skills described here.

Quick Facts

- Most math questions will be multiple choice, but some will be student-produced responses (grid-ins).
- The Math Test is divided into two portions: Math Test – Calculator and Math Test – No Calculator.
- Some parts of the test present students with a scenario and then ask several questions about it.

Distinctive Features

The Math Test is characterized by questions that:

- Test mathematical reasoning in a way that reflects the work students are doing in classrooms across the country.
- Emphasize fluency and understanding.
- Ask students to solve problems grounded in science, social science, career scenarios, and other real-life contexts.

The test covers all math practices, with an emphasis on problem solving, modeling, using appropriate tools strategically, and recognizing and using algebraic structure.

Student-Produced Response Questions

Although most of the questions on the Math Test are multiple choice, a percentage of the questions — from 17 percent to 22 percent, depending on the assessment — are student-produced response questions, also known as grid-ins. Instead of choosing a correct answer from a list of options, students are required to solve problems and enter their answers in the grids provided on the answer sheet.

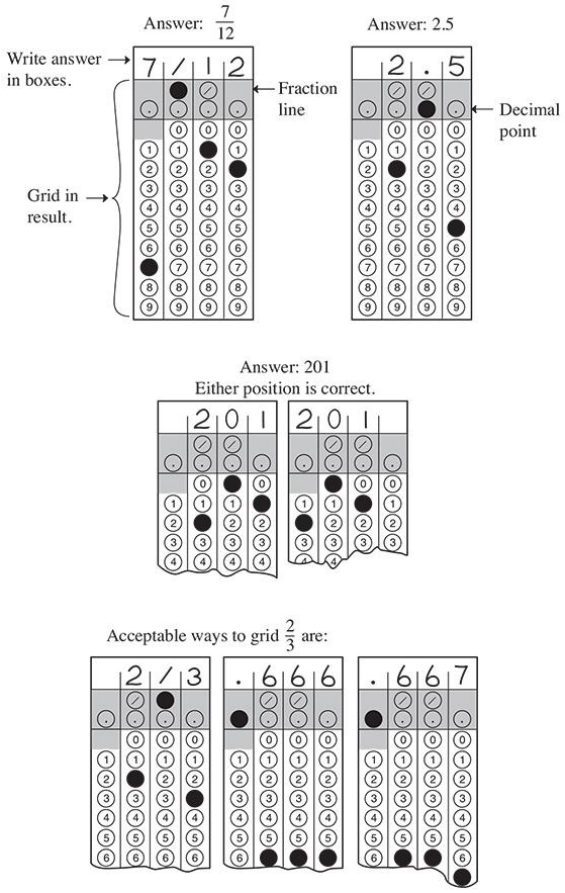
Sometimes students will be asked several questions about the same scenario, allowing them to dig in to a situation and model it mathematically. These multistep applications reflect the complexity of real-life problem solving in science, social science, and career contexts.

Note that the MESA Day contest will NOT include student-produced response questions.

Gridding-In Answers

- Mark no more than one circle in any column.
- Only answers indicated by filling in the circle will be scored (students do not receive credit for anything written in the boxes located above the circles).
- It doesn't matter in which column students begin entering their answers; as long as the responses are recorded within the grid area, students will receive credit.
- The grid can hold only four decimal places and can only accommodate positive numbers and zero.
- Unless a problem indicates otherwise, answers can be entered on the grid as a decimal or a fraction.
- Fractions like $\frac{3}{24}$ do not need to be reduced to their lowest terms.
- All mixed numbers need to be converted to improper fractions before being recorded in the grid.
- If the answer is a repeating decimal, students must grid the most accurate value the grid will accommodate.

Below is a sample of the instructions students will see on the test.



Calculator Use

Students will be allowed to use a calculator on one of two portions of the Math Test. Calculators are important mathematical tools, and to succeed after high school, students have to know how to use them effectively and appropriately.

In the Math Test – Calculator portion of the test, students can use their calculator to make computations more efficiently, enabling them to focus on complex modeling and reasoning. However, the calculator is a tool that students must use strategically, deciding when to use it — and when not to. There will be some Calculator: Permitted questions that can be answered more efficiently without a calculator. In these cases, students who make use of structure or their ability to reason will most likely reach the solution more rapidly than students who use a calculator.

The Math Test – No Calculator portion of the test makes it easier to assess students' fluency in math and their understanding of some math concepts. It also tests well-learned technique and number sense.

Components of Mathematical Proficiency

The Math Test assesses fluency with mathematical procedures, conceptual understanding, and applications with equal intensity, as they are the primary components of mathematical proficiency.

Fluency

For the Math Test, "fluency" is a skill in carrying out procedures flexibly, accurately, and efficiently with strategic competence. That is, students are expected to demonstrate a reasonable quickness when solving problems by identifying and using the most efficient solution approaches, such as solving a problem by inspection, using their mathematical understanding and skills to find a shortcut, or reorganizing the information given.

Conceptual Understanding

The Math Test requires students to demonstrate conceptual understanding by demonstrating mastery of mathematical concepts, operations, and relations. For example, questions may require making connections between properties of linear equations, their graphs, and the contexts they represent.

Applications

Applications on the Math Test require students to demonstrate the ability to analyze a situation, determine the essential elements required to solve the problem, represent the problem mathematically, and carry out a solution. Application problems are set in the real world. Many of these problems are set in academic and career settings and are likely to draw from the sciences and social sciences.

Scoring Changes

The redesigned SAT will be scored differently from the way it's been scored in the past. A few changes:

- The test will share a common score scale with the other tests in the SAT Suite because the content is so tightly aligned.
- Subscores and cross-test scores will be added for greater insight.
- There won't be a penalty for guessing. The SAT Suite of Assessments will not deduct points for incorrect answers. Students will earn points for the questions they answer correctly. This move to rights-only scoring encourages students to give the best answer they have to every question.

Benefit to Society

Many students ask why study math. Others say “Who cares about math and when am I ever going to need it?”

Mathematician John Allen Paulos writes,

"As a mathematician, I'm often challenged to come up with compelling reasons to study mathematics. If the questioner is serious, I reply that there are three reasons or, more accurately, three broad classes of reasons to study mathematics. Only the first and most basic class is practical. It pertains to job skills and the needs of science and technology. The second concerns the understandings that are essential to an informed and effective citizenry. The last class of reasons involves considerations of curiosity, beauty, playfulness, perhaps even transcendence and wisdom." [Paulos, John Allen. *A Mathematician Reads the Newspaper,* p164-168. Basic Books (1995)]

Broadly speaking, studying math is critical because it is:

- Useful:
  - Mathematical problems abound in daily life
  - Mathematical proficiency is required for many jobs
  - Mathematics is essential for science, engineering, and research

- Important:
  - A mathematically informed citizenry will make better economic and political decisions about risk, policy, and resource allocation

- Interesting:
  - "Mathematics, rightly viewed, possesses not only truth, but supreme beauty" (Bertrand Russell) and should be studied in its own right
  - The landmark accomplishments of mathematics stand alongside the masterworks of art and music as cultural triumphs that all educated persons should be able to appreciate
  - Doing mathematics teaches patterns of problem-solving and insight that transfer to other knowledge domains
  - Mathematical proof teaches skills in rigor, argumentation and persuasion that transfer to other knowledge domains

Mathematics has been a part of a traditional western education since classical times -- the door to Plato's Academy in ancient Greece reportedly bore the inscription "Let No One Ignorant of Geometry Enter Here." In *The Republic*, Plato lays out a course of study for citizens of an ideal society. His higher education begins with Arithmetic, of which "all arts and sciences necessarily partake," and which "leads to the apprehension of truth:" [Plato. *The Republic*, Book VII (360 B.C.E) tr. Benjamin Jowett (1901)]

SAT Math Domains / Module Content

Lesson # 1 – The Heart of Algebra

Heart of Algebra is one of the three SAT Suite of Assessment Math subscores, reported on a scale of 1 to 15. Heart of Algebra ask students to:

1. **Create, solve, or interpret a linear expression or equation in one variable** that represents a context. The expression or equation will have rational coefficients, and multiple steps may be required to simplify the expression, simplify the equation, or solve for the variable in the equation.
2. **Create, solve, or interpret linear inequalities in one variable** that represent a context. The inequality will have rational coefficients, and multiple steps may be required to simplify or solve for the variable.
3. **Build a linear function that models a linear relationship between two quantities.** The student will describe a linear relationship that models a context using either an equation in two variables or function notation. The equation or function will have rational coefficients, and multiple steps may be required to build and simplify the equation or function.
4. **Create, solve, and interpret systems of linear inequalities in two variables.** The student will analyze one or more constraints that exist between two variables by creating, solving, or interpreting an inequality in two variables or a system of inequalities in two variables to represent a context. Multiple steps may be required to create the inequality or system of inequalities or to determine whether a given point is in the solution set.
5. **Create, solve, and interpret systems of two linear equations in two variables.** The student will analyze one or more constraints that exist between two variables by creating, solving, or analyzing a system of linear equations to represent a context. The equations will have rational coefficients, and multiple steps may be required to simplify or solve the system.
6. **Algebraically solve linear equations (or inequalities) in one variable.** The equation (or inequality) will have rational coefficients and may require multiple steps to solve for the variable; the equation may yield no solution, one solution, or infinitely many solutions. The student may also be asked to determine the value of a constant or coefficient for an equation with no solution or infinitely many solutions.
7. **Algebraically solve systems of two linear equations in two variables.** The equations will have rational coefficients, and the system may yield no solution, one solution, or infinitely many solutions. The student may also be asked to determine the value of a constant or coefficient of an equation in which the system has no solution, one solution, or infinitely many solutions.
8. **Interpret the variables and constants in expressions for linear functions within the context presented.** The student will make connections between a context and the linear equation that models the context and will identify or describe the real-life meaning of a constant term, a variable, or a feature of the given equation.

9. **Understand connections between algebraic and graphical representations.** The student will select a graph described by a given linear equation, select a linear equation that describes a given graph, determine the equation of a line given a verbal description of its graph, determine key features of the graph of a linear function from its equation, or determine how a graph may be affected by a change in its equation.

   Example 1:

   If $\frac{1}{2}x + \frac{1}{3}y = 4$, what is the value of $3x + 2y$?

   A student may find the solution to this Heart of Algebra problem by noticing the structure of the given equation and seeing that multiplying both sides of the equation $\frac{1}{2}x + \frac{1}{3}y = 4$ by 6 to clear the fractions from the equation yields $3x + 2y = 24$.

   Example 2:

   $4x - y = 3y + 7$

   $x + 8y = 4$

   A) $\frac{3}{2}$

   B) $\frac{1}{4}$

   C) $\frac{1}{2}$

   D) $\frac{11}{9}$

   Example 2, again from Heart of Algebra, rewards fluency in solving pairs of simultaneous linear equations. Rather than looking for a clever way of back solving the value of the product *xy* from system, students can solve the system for the values of x and y, then simply multiply them to get choice C) $\frac{1}{2}$. Note that because the system is not given in standard form, this requires doing some additional algebra, further reinforcing the need for fluency.

For additional sample problems, please see the College Board "Test Specifications for the Redesigned SAT:"

- Pages 156 – 167 (Samples Questions 1 to 10: Heart of Algebra)

Lesson # 2 – Problem Solving and Data Analysis

Problem Solving and Data Analysis is one of the three SAT Suite of Assessment Math subscores, reported on a scale of 1 to 15. Problem Solving and Data Analysis questions ask students to:

1. **Use ratios, rates, proportional relationships, and scale drawings to solve single- and multistep problems.** The student will use a proportional relationship between two variables to solve a multistep problem to determine a ratio or rate; calculate a ratio or rate and then solve a multistep problem; or take a given ratio or rate and solve a multistep problem.
2. **Solve single- and multistep problems involving percentages.** The student will solve a multistep problem to determine a percentage; calculate a percentage and then solve a multistep problem; or take a given percentage and solve a multistep problem.
3. **Solve single- and multistep problems involving measurement quantities, units, and unit conversion.** The student will solve a multistep problem to determine a unit rate; calculate a unit rate and then solve a multistep problem; solve a multistep problem to complete a unit conversion; solve a multistep problem to calculate density; or use the concept of density to solve a multistep problem.
4. **Given a scatterplot, use linear, quadratic, or exponential models to describe how the variables are related.** The student will, given a scatterplot, select the equation of a line or curve of best fit; interpret the line in the context of the situation; or use the line or curve of best fit to make a prediction.
5. **Use the relationship between two variables to investigate key features of the graph.** The student will make connections between the graphical representation of a relationship and properties of the graph by selecting the graph that represents the properties described, or using the graph to identify a value or set of values.
6. **Compare linear growth with exponential growth.** The student will infer the connection between two variables given a context in order to determine what type of model fits best.
7. **Use two-way tables to summarize categorical data and relative frequencies, and calculate conditional probability.** The student will summarize categorical data or use categorical data to calculate conditional frequencies, conditional probabilities, association of variables, or independence of events.
8. **Make inferences about population parameters based on sample data.** The student will estimate a population parameter given the results from a random sample of the population. The sample statistics may mention confidence intervals and measurement error that the student should understand and make use of, but need not calculate.
9. **Use statistics to investigate measures of center of data and analyze shape, center, and spread.** The student will calculate measures of center and/or spread for a given set of data or use given statistics to compare two separate sets of data. The measures of center that may be calculated include mean, median, and mode, and the measures of spread that may be calculated include range. When comparing two data sets, the student may investigate mean, median, mode, range, and/or standard deviation.

10. **Evaluate reports to make inferences, justify conclusions, and determine appropriateness of data collection methods.** The reports may consist of tables, graphs, or text summaries.

For sample problems, please see the College Board "Test Specifications for the Redesigned SAT:"

- Pages 168 – 177 (Sample Questions 11 to 16: Problem Solving and Data Analysis)

Lesson # 3 – Passport to Advanced Math

Passport to Advanced Math is one of the three SAT Suite of Assessment Math subscores, reported on a scale of 1 to 15. Passport to Advanced Math questions ask students to:

1. **Create a quadratic or exponential function** or equation that models a context. The equation will have rational coefficients and may require multiple steps to simplify or solve the equation.
2. **Determine the most suitable form of an expression** or equation to reveal a particular trait, given a context.
3. **Create equivalent expressions involving rational exponents** and radicals, including simplifying or rewriting in other forms.
4. **Create an equivalent form of an algebraic expression** by using structure and fluency with operations.
5. **Solve a quadratic equation** having rational coefficients. The equation can be presented in a wide range of forms to reward attending to algebraic structure and can require manipulation in order to solve.
6. **Add, subtract, and multiply polynomial expressions** and simplify the result. The expressions will have rational coefficients.
7. **Solve an equation in one variable that contains radicals or contains the variable in the denominator of a fraction.** The equation will have rational coefficients, and the student may be required to identify when a resulting solution is extraneous.
8. **Solve a system of one linear equation and one quadratic equation.** The equations will have rational coefficients.
9. **Rewrite simple rational expressions.** Students will add, subtract, multiply, or divide two rational expressions or divide two polynomial expressions and simplify the result. The expressions will have rational coefficients.
10. **Interpret parts of nonlinear expressions in terms of their context.** Students will make connections between a context and the nonlinear equation that models the context to identify or describe the real-life meaning of a constant term, a variable, or a feature of the given equation.
11. **Understand the relationship between zeros and factors of polynomials**, and use that knowledge to sketch graphs. Students will use properties of factorable polynomials to solve conceptual

problems relating to zeros, such as determining whether an expression is a factor of a polynomial based on other information provided.

12. **Understand a nonlinear relationship between two variables** by making connections between their algebraic and graphical representations. The student will select a graph corresponding to a given nonlinear equation; interpret graphs in the context of solving systems of equations; select a nonlinear equation corresponding to a given graph; determine the equation of a curve given a verbal description of a graph; determine key features of the graph of a linear function from its equation; or determine the impact on a graph of a change in the defining equation.
13. **Use function notation, and interpret statements using function notation.** The student will use function notation to solve conceptual problems related to transformations and compositions of functions.
14. **Use structure to isolate or identify a quantity of interest** in an expression or isolate a quantity of interest in an equation. The student will rearrange an equation or formula to isolate a single variable or a quantity of interest.

Example 1

The function *f* is defined by $f(x) = 2x^3 + 3x^2 + cx + 8$, where *c* is a constant. In the *xy*-plane, the graph of *f* intersects the *x*-axis at the three points (–4, 0), $(\frac{1}{2}, 0)$, and (*p*, 0). What is the value of *c*?

A) –18

B) –2

C) 2

D) 10

Example 1, from Passport to Advanced Math, assesses conceptual understanding of polynomials and their graphs. If a student understands these concepts and requires, for example, the point (–4, 0) to lie on the graph, this results in $0 = 2(-4)^3 + 3(-4)^2 + c(-4) + 8$. A student who looks for and makes use of structure will monitor the calculation at this point and recognize an equation that determines the desired value of *c*, –18. Seeing that he or she is on the right track, the student will then perform the calculations required to solve for *c*.

For additional sample problems, please see the College Board "Test Specifications for the Redesigned SAT:"

- Pages 178 – 186 (Samples Questions 17 to 23: Passport to Advanced Math)

Lesson # 4 – Additional Topics in Math

Additional topics will include geometric and trigonometric concepts. The questions in this domain contribute to the Math area score (200-800) but not to any specific math subscore. Additional Topics in Math questions asks students to:

1. **Solve problems using volume formulas.** The student will use given information about figures, such as length of a side, area of a face, or volume of a solid, to calculate missing information. Any required volume formulas will be provided to students either on the formula sheet or within the question.
2. **Use trigonometric ratios and the Pythagorean Theorem** to solve applied problems involving right triangles. The student will use information about triangle side lengths or angles presented in a context to calculate missing information using the Pythagorean Theorem and/or trigonometric ratios.
3. Add, subtract, multiply, divide, and simplify **complex numbers**.
4. **Convert between degrees and radians and use radians to determine arc lengths; use trigonometric functions of radian measure.** The student will convert between angle measures in degrees and radians in order to calculate arc lengths by recognizing the relationship between an angle measured in radians and an arc length, evaluating trigonometric functions of angles in radians.
5. **Apply theorems about circles to find arc lengths, angle measures, chord lengths, and areas of sectors.** The student will use given information about circles and lines to calculate missing values for radius, diameter, chord length, angle, arc, and sector area.
6. **Use concepts and theorems about congruence and similarity to solve problems about lines, angles, and triangles.** The student will use theorems about triangles and intersecting lines to determine missing lengths and angle measures of triangles. The student may also be asked to provide a missing length or angle to satisfy a given theorem.
7. **Use the relationship between similarity, right triangles, and trigonometric ratios; use the relationship between sine and cosine of complementary angles.** The student will use trigonometry and theorems about triangles and intersecting lines to determine missing lengths and angle measures of right triangles. The student may also be asked to provide a missing length or angle that would satisfy a given theorem.
8. **Create or use an equation in two variables to solve a problem about a circle in the coordinate plane.** The student will create an equation or use properties of an equation of a circle to demonstrate or determine a property of the circle's graph.

For sample problems, please see the College Board "Test Specifications for the Redesigned SAT:"

- Pages 187 – 191 (Samples Questions 24 to 26: Additional Topics in Math)

Lesson # 5: Practice on Your Own

The most important path to college success has always been excellent work in the classroom. But the rise of costly test preparation has introduced a perception of inequality among students. With the redesign of the SAT, the College Board, in partnership with Khan Academy, is announcing its commitment to providing free, high-quality SAT practice materials for all.

Try Sample Questions

To help prepare for the new SAT (first offered in spring 2016), the College Board has released a significant number of test questions and made them available on its website.

Review Sample Questions
(https://collegereadiness.collegeboard.org/sample-questions)

Personalized Study for the SAT with Khan Academy

Students who take any test in the SAT Suite of Assessments — or an official SAT Practice Test online at KhanAcademy.org — will have access to personalized SAT study resources based on their results. The Khan Academy practice experience will be adaptive and tailored to meet each student's individual needs. To ensure that all test-takers have access to practice, the College Board and Khan Academy are collaborating with community-based organizations, like the Boys & Girls Clubs of America, as well as teachers and schools.

The following practice resources are available now on Khan Academy:

- Thousands of practice questions, reviewed and approved by the College Board
- The four official SAT Practice Tests written by the College Board
- Personalized recommendations for instruction and practice to help students fill their knowledge gaps

Practice on Khan Academy
(https://www.khanacademy.org/sat)