#3

Learning Activity #3:

Analyze and Evaluate a Truss

Overview of the Activity

In this learning activity, we will analyze and evaluate one of the main trusses from the Grant Road Bridge. We will create a mathematical model of the truss, then use this model as the basis for a structural analysis—a series of mathematical calculations to determine the internal force in every member of the truss. We will also use the experimental data from Learning Activity #2 to determine the strength of each truss member. Finally we will perform a structural evaluation—a comparison of the internal forces and strengths, to determine whether or not the truss can safely carry its prescribed loads.

Why?

Engineering design is an iterative process. To create an optimal design, the engineer must develop many different alternative solutions, evaluate each one, and then select the alternative that best satisfies the design requirements. But how are these alternative solutions evaluated? Engineers use many different criteria to evaluate a design; but in structural design, the most important of these criteria is the structure's ability to carry load safely. In most cases, an evaluation of structural safety can only be done mathematically. It would be impractical, uneconomical, and unsafe for the structural engineer to evaluate a bridge design by building a full-size prototype, then running heavy trucks across the structure to determine if it is strong enough. When a structure is built, it must be strong enough to carry its prescribed loads. The engineer must get it right the first time. For this reason, the structural engineer must be able to mathematically model, analyze, and evaluate the structure with a high degree of accuracy—and without the benefit of prototype testing. In this activity, you will learn how an engineer performs a structural evaluation. In Learning Activity #5, you will apply this process to design your own truss bridge.

Learning Objectives

As a result of this learning activity, you will be able to do the following:

- Calculate the components of a force vector.
- Add two force vectors together.
- Explain the following structural engineering concepts: *free body diagram, equilibrium, structural model, symmetry, static determinacy, stability, and factor of safety.*
- Use the Method of Joints to calculate the internal force in every member in a truss.
- Determine the strength of every member in a truss.
- Evaluate a truss, to determine if it can carry a given load safely.

Key Terms

To successfully complete this learning activity, you must understand the following key terms and concepts from Learning Activities #1 and #2:

truss	**deck**	**internal force**	**tensile strength**
member	**load**	**tension**	**compressive strength**
joint	**reaction**	**compression**	**failure**

If you have forgotten any of these terms, it would be a good idea to review their definitions in the Glossary (Appendix D) before proceeding.

Information

Analysis

An **analysis** is an examination of a complex system, usually conducted by breaking the system down into its component parts. Once they are identified, the component parts and their relationships to the system as a whole can be studied in detail. For example, suppose your baseball team has been losing a lot of games, and you want to figure out why. Your team is a complex system. There are a lot of possible reasons why it might not be functioning as well as it could. To analyze the performance of the team, you'll need to break it down into its component parts. The obvious way to do this is to look at the team's individual members—nine players and a coach. But the team can also be broken down by its *functions*—hitting, pitching, fielding, and base running. To perform the analysis, you would look at each team member and each function in detail. You would examine batting, pitching, and fielding statistics, to determine whether poor performance in any of these areas might be responsible for the team's losing record. You might discover, for example, that the team's batting average against left-handed pitching has been particularly poor. This important analysis result might be used as the basis for designing a practice regimen to correct the problem.

Structural Analysis

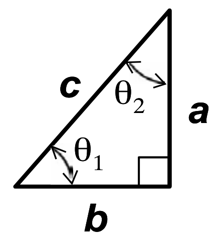
A **structural analysis** is a mathematical examination of a structure, conducted by breaking the structure down into its component parts, then studying how each part performs and how each part contributes to the performance of the structure as a whole. Usually, the products of a structural analysis are (1) reactions, (2) internal member forces, and (3) deflections—how much the structure bends or sways when it is loaded. Like the analysis of your baseball team, structural analysis is often used to determine if the system is performing as intended and, if it is not, to correct the problem.

There is an important difference between *structural analysis and structural design*. Structural analysis is concerned with examining *existing structures* to determine if they can carry load safely. Structural design is concerned with creating *new structures* to meet the needs of society. Though analysis and design are fundamentally different activities, they are closely interrelated—analysis is an integral part of the design process. We'll see how analysis and design fit together in Learning Activity #4.

To perform a structural analysis, we will apply a variety of mathematical tools from geometry, trigonometry, and algebra, as well as some basic concepts from physics. These concepts are reviewed in the following sections.

Some Basic Concepts from Trigonometry

A truss is a structure composed of members arranged in interconnected triangles. For this reason, the geometry of triangles is very important in structural analysis. To analyze a truss, we must be able to mathematically relate the angles of a triangle to the lengths of its sides. These relationships are part of a branch of mathematics called **trigonometry**. Here we will review some basic concepts from trigonometry that are essential tools for truss analysis.



This diagram shows a **right triangle**—a triangle with one of its three angles measuring exactly 90°. Sides ***a*** and ***b*** form the 90° angle. The other two angles, identified as θ_1 and θ_2, are always less than 90°. Side ***c***, the side opposite the 90° angle, is always the longest of the three sides. It is called the **hypotenuse** of the right triangle.

Thanks to an ancient Greek mathematician named Pythagoras, we can easily calculate the length of the hypotenuse of a right triangle. The **Pythagorean Theorem** tells us that

$$c = \sqrt{a^2 + b^2}$$

The Pythagorean Theorem shows how the lengths of the sides of a right triangle are related to each other. But how are the lengths of the sides related to the angles? Consider the definitions of two key terms from trigonometry—*sine* and *cosine*. Both definitions are based on the geometry of a right triangle, as shown above. The **sine** of an angle (abbreviated ***"sin"***) is defined as the length of the *opposite side* divided by the length of the *hypotenuse*. For example, the sine of the angle θ_1 would be calculated as

$$\sin\theta_1 = \frac{opposite}{hypotenuse} = \frac{a}{c}$$

In this case, side ***a*** is designated as the "opposite side," because it is farthest from the angle θ_1. For the angle θ_2, the opposite side is ***b***; thus, the sine of θ_2 is

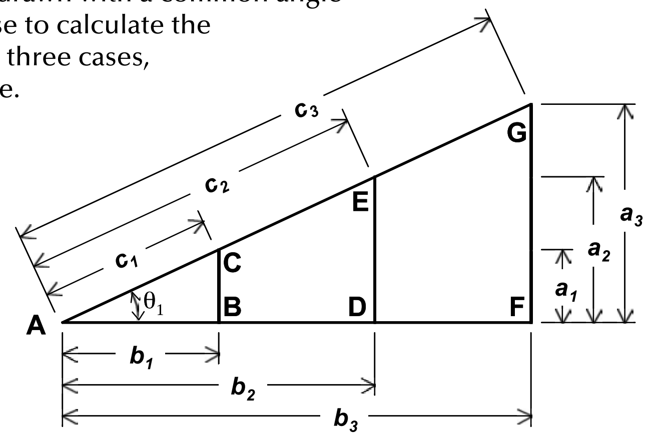
$$\sin\theta_2 = \frac{opposite}{hypotenuse} = \frac{b}{c}$$

The **cosine** of an angle (abbreviated ***"cos"***) is defined as the length of the *adjacent side* divided by the length of the *hypotenuse*. Applying this definition to our example, we have

$$\cos\theta_1 = \frac{adjacent}{hypotenuse} = \frac{b}{c}$$

$$\cos\theta_2 = \frac{adjacent}{hypotenuse} = \frac{a}{c}$$

It is important to recognize that the sine and cosine of an angle do not depend on the overall size of the triangle—only on the relative lengths of its sides. In the diagram at right, three different right triangles (ABC, ADE, and AFG) are drawn with a common angle θ_1. It doesn't matter which of the three triangles you use to calculate the sine and cosine of θ_1. You'll get the same answers in all three cases, because the relative lengths of the sides are all the same.



$$\sin\theta_1 = \frac{a_1}{c_1} = \frac{a_2}{c_2} = \frac{a_3}{c_3}$$

$$\cos\theta_1 = \frac{b_1}{c_1} = \frac{b_2}{c_2} = \frac{b_3}{c_3}$$

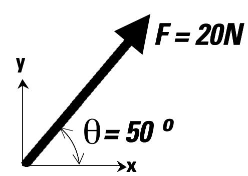
We'll see important applications of the sine and cosine when we analyze a truss, later in this learning activity.

Working with Vectors

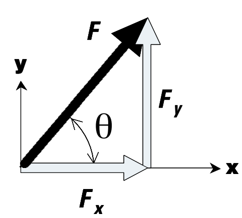
A force can be represented as a **vector**—a mathematical quantity that has both magnitude and direction. When we perform a structural analysis, we will calculate both the magnitude and direction of every force that acts on the structure. Thus, before when can analyze a structure, we need to learn how to work with vectors. Specifically, we need to learn two basic concepts from vector math—breaking a vector into its components and adding vectors together.

Breaking a Vector into its Components

When we analyze a truss, we will need to describe the *directions* of force vectors mathematically. To do this, we must first define a *coordinate axis system*. For a two-dimensional structure, we normally use an x-axis to represent the horizontal direction and a y-axis to represent the vertical. Once the coordinate axis system is established, we can represent the direction of any vector as an *angle* measured from either the x-axis or the y-axis. For example, the force vector at right has a magnitude *(**F**)* of 20 newtons and a direction *(*θ) of 50 degrees, measured counterclockwise from the x-axis.



This force can also be represented as *two equivalent forces*, one in the x-direction and one in the y-direction. Each of these forces is called a **component** of the vector ***F***. To determine the magnitudes of these two components, visualize a right triangle with the vector ***F*** as the hypotenuse and the other two sides parallel to the x-axis and y-axis. If ***F*** is the length of the hypotenuse, then the lengths of the two perpendicular sides are exactly equal to the x-component and y-component of ***F***. We use the symbol F_x to represent the x-component of ***F*** and the symbol F_y to represent the y-component.



From trigonometry, we can apply the definitions of the sine and the cosine to calculate the two components. Recall that

$$\sin\theta = \frac{opposite}{hypotenuse}$$

From the diagram on the previous page, we can see that $\boldsymbol{F_y}$ is the opposite side of the triangle, and $\boldsymbol{F}$ is the hypotenuse. Substituting, we get

$$\sin\theta = \frac{opposite}{hypotenuse} = \frac{\boldsymbol{F_y}}{\boldsymbol{F}}$$

If we multiply both sides of this equation by $\boldsymbol{F}$, we get

$$\underline{\underline{\boldsymbol{F_y} = \boldsymbol{F}\sin\theta}}$$

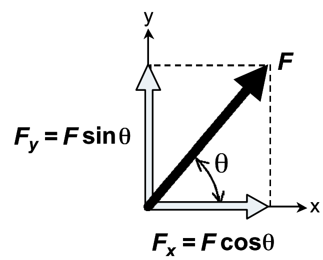
Similarly,

$$\cos\theta = \frac{adjacent}{hypotenuse} = \frac{\boldsymbol{F_x}}{\boldsymbol{F}}$$

$$\underline{\underline{\boldsymbol{F_x} = \boldsymbol{F}\cos\theta}}$$

Therefore, if we know the magnitude *(**F**)* and direction *(θ)* of a force, then we can use the equations above to calculate the two components of the force.

The diagram at right shows the correct way to represent the force $\boldsymbol{F}$ and its components—with all three vectors originating from the same point. The two dotted lines show that $\boldsymbol{F_x}$ and $\boldsymbol{F_y}$ are the same lengths as the sides of a right triangle with $\boldsymbol{F}$ as its hypotenuse.



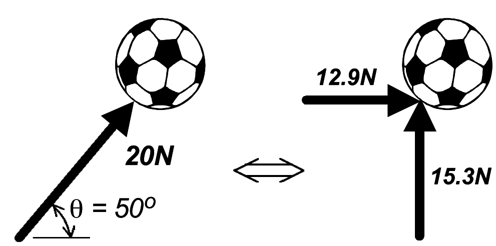
Returning to our example, if we substitute the actual numerical values F=*20N* and θ=50°, and use a calculator to determine the sine and cosine of the angle, we get the following results

$$\boldsymbol{F_y} = \boldsymbol{F}\sin\theta = \boldsymbol{20}\sin\boldsymbol{50}^\circ = \boldsymbol{20}(\boldsymbol{0.766}) = \underline{\underline{\boldsymbol{15.3}N}}\uparrow$$

$$\boldsymbol{F_x} = \boldsymbol{F}\cos\theta = \boldsymbol{20}\cos\boldsymbol{50}^\circ = \boldsymbol{20}(\boldsymbol{0.643}) = \underline{\underline{\boldsymbol{12.9}N}}\rightarrow$$

The small arrows to the right of the answers indicate the directions of the $\boldsymbol{F_y}$ and $\boldsymbol{F_x}$ vectors. When we write a vector quantity, we must always be careful to show *both* its magnitude and direction.

But what do these numbers really mean? Suppose you kick a soccer ball with a single 20-newton force at an angle of 50°. This force will cause the ball to move a particular direction and distance. Now suppose that two players kick the ball simultaneously—one with a 15.3-newton force in the y-direction and one with a 12.9-newton force in the x-direction. In this case, the ball will respond exactly as it did when you kicked it with the single 20-newton force. The ball will move the same direction and distance, because it "feels" exactly the same force. The two components of a force are exactly equivalent to that force and will produce exactly the same effect on an object.



The two components of a force are exactly equivalent to that force.

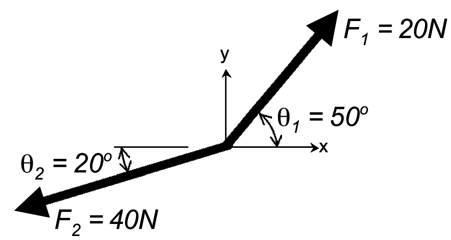
Adding Vectors Together

When two or more forces are applied to an object, it is often necessary to calculate the *total force* on the object. We calculate the total force by simply adding all of the individual force vectors together. To add vectors, however, we must follow an important rule:

The magnitudes of two or more vectors can be added together only if their directions are the same.

To add vectors whose directions are not the same, we must do the following:

- Break each vector into its equivalent x-component and y-component.
- Add all of the x-components together.
- Add all of the y-components together.



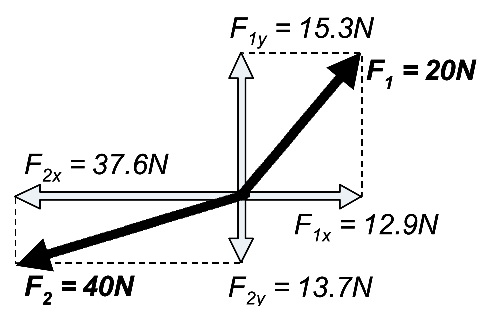
As an example, let's add the two forces $\mathbf{F_1}$ and $\mathbf{F_2}$, shown at right. We begin by calculating the components of the two vectors:

$$F_{1x} = F_1 \cos\theta_1 = 20 \cos 50^\circ = 20(0.643) = 12.9N \rightarrow$$

$$F_{1y} = F_1 \sin\theta_1 = 20 \sin 50^\circ = 20(0.766) = 15.3N \uparrow$$

$$F_{2x} = F_2 \cos\theta_2 = 40 \cos 20^\circ = 40(0.940) = 37.6N \leftarrow$$

$$F_{2y} = F_2 \sin\theta_2 = 40 \sin 20^\circ = 40(0.342) = 13.7N \downarrow$$



Again the direction of each vector component is indicated with an arrow. We must pay careful attention to these directions when we add components together. Note that F_{1x} and F_{2x} point in opposite directions. The directions of F_{1y} and F_{2y} are also opposite.

When we add the x-components, we will assume that the direction indicated by the x-axis is positive. Then the sum of the two x-components is

$$F_{TOTALx} = F_{1x} - F_{2x} = +12.9 - 37.6 = -24.7N$$

In this equation, F_{1x} is positive, because it points to the right—the same direction as the positive x-axis. F_{2x} is negative, because it points to the left—opposite the direction of the positive x-axis. The answer is negative, which means that the x-component of the total force is to the left. We write the final answer as

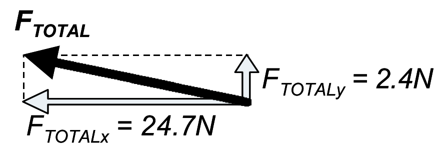
$$F_{TOTALx} = \underline{\underline{24.7N}} \leftarrow$$

Assuming that the direction of the positive y-axis (upward) is positive, the sum of the y-components is

$$F_{TOTALy} = F_{1y} - F_{2y} = +15.3 - 12.9 = +2.4N = \underline{\underline{2.4N}} \uparrow$$

In this case, the total is positive, so we conclude that the y-component of the total force is upward.

The total force and its two components are illustrated at right. If we needed to know the actual magnitude of F_{TOTAL} we could calculate it by using the Pythagorean Theorem; however, for this learning activity, we will only need to calculate the total x-component and the total y-component, as shown here.



Equilibrium

In Learning Activity #1, we defined *equilibrium* as a condition in which the total force acting on an object is zero. Now that we know how to actually calculate the total force on an object, we can apply the concept of equilibrium as a powerful problem-solving tool. Specifically, if we know that an object is in equilibrium—because it is not moving—then we know that the total force on that object is zero; and we can use this fact to calculate the magnitude and direction of unknown forces acting on the object.

Because we calculate total force by adding up the x-components and y-components separately, there are really two conditions that must be satisfied if an object is in equilibrium.*

First, the sum of the x-components of all forces acting on the structure must be zero. We write this condition as

$$\sum F_x = 0$$

where the symbol Σ means "the sum of," and the entire expression is read, "The sum of the forces in the x-direction equals zero."

The second equilibrium condition is that the sum of all forces in the y-direction must equal zero, which we write as

$$\sum F_y = 0$$

These two equations are commonly known as the **equations of equilibrium**. They are simple yet powerful mathematical tools, with many different applications in science and engineering. In this learning activity, the equations of equilibrium will enable us to calculate the reactions and internal member forces in a truss.

Creating a Structural Model

A **structural model** is a mathematical idealization of a structure—a series of simplifying assumptions about the structure's configuration and loading that allow us to predict its behavior mathematically.

When we model a two-dimensional truss, we typically make the following general assumptions:

- The truss members are perfectly straight.
- The joints that connect the truss members together are frictionless pins.
- Loads and reactions are applied only at the joints.

* Actually, there are three equilibrium conditions for a two-dimensional structure. In addition to the two described above, the sum of the *moments* about any point must also equal zero. The concept of a moment is a very important one; however, it is beyond the scope of this book. The problems used in this and subsequent learning activities have been chosen so that this third equilibrium condition is not required to obtain a correct solution.

Taken together, these assumptions imply that *the members of a truss do not bend*. Truss members are assumed to carry load either in pure *tension* or in pure *compression*. These assumptions allow us to use a simple type of structural analysis that ignores the effects of bending.

None of these assumptions is perfectly accurate, however. In an actual truss bridge, members are never perfectly straight, due to minor variations in the manufacturing and fabrication processes. Modern trusses use gusset plate connections, which do not behave like pins; and even in older bridges with pinned connections, the pins are certainly not frictionless. Furthermore, actual trusses can never be loaded entirely at the joints, if only because the weight of the members themselves is distributed throughout the structure. Fortunately, the inaccuracies in our assumptions generally produce only minor inaccuracies in our structural analysis results, and experienced engineers know how to compensate for these small errors to ensure the safety of their designs.

Having made these general assumptions about the structure, we must also make a number of specific decisions about how to represent the particular truss we are modeling. These decisions include:

- The geometric configuration of the truss, including the locations of all joints, the configuration of the members, and all relevant dimensions.
- The configuration of the supports.
- The magnitude and direction of the loads that will be applied to the structure.

Once we have decided how we will represent the structure, supports, and loads, we should always complete the modeling process by creating one or more drawings that clearly illustrate the structural model.

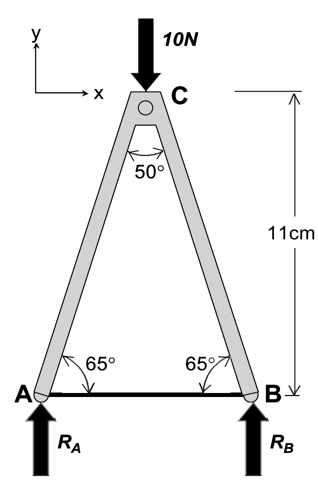
The Free Body Diagram

One of the most important tools in structural engineering is a simple sketch called the *free body diagram*. A **free body diagram** is a drawing of a "body"—a structure or a portion of a structure—showing all of the forces acting on it. Drawing a free body diagram is the essential first step in any structural analysis.

To draw a free body diagram:

- Draw the outline of the structure, completely isolated from its surroundings. Do not show any of the supports that connect the structure to its foundations.
- At the location of each support, draw and label the appropriate *reactions*.
- Draw and label all of the loads applied to the structure.
- Draw all relevant dimensions.
- Draw the x-y coordinate axis system.

In Learning Activity #1, we built a simple three-member truss by tying a short piece of string to the handles of a nutcracker. We then applied a 10-newton downward load to the top of the structure. The free body diagram for our nutcracker truss is shown here. Note that the downward 10-newton load is resisted by two upward reactions at the bottom ends of the handles. Because the magnitudes of these forces are unknown, they are labeled R_A and R_B. The three joints of the truss are labeled **A**, **B**, and **C**, for future reference.



Free Body Diagram of the nutcracker truss

Calculating Reactions

Reactions are forces developed at the supports of a structure, to keep the structure in equilibrium. Given that the reactions R_A and R_B on our nutcracker truss are in the y-direction, we can determine their magnitude using the equilibrium equation

$$\sum F_y = 0$$

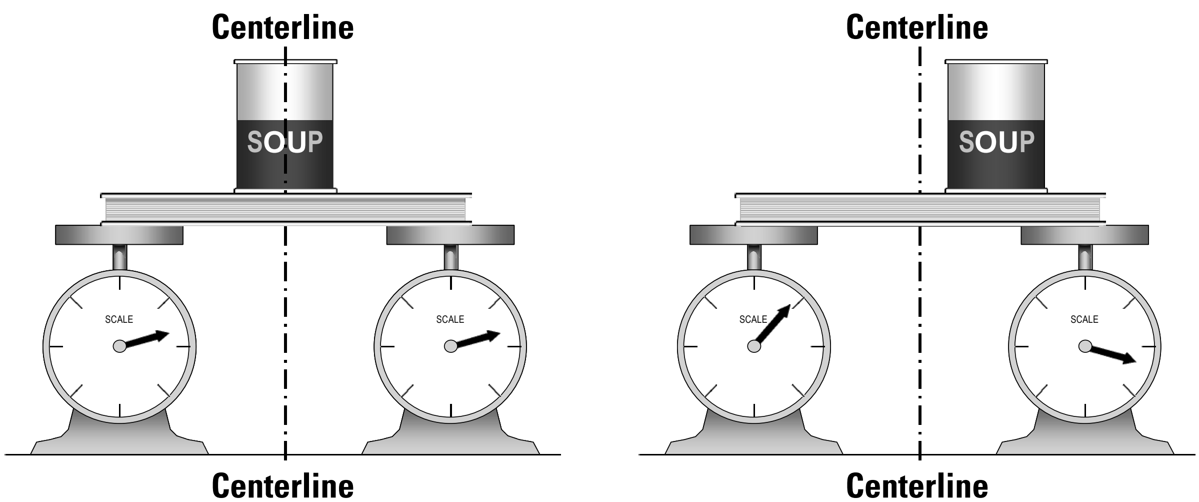
Assuming that the upward direction is positive, the sum of the forces in the y-direction is

$$\sum F_y = R_A + R_B - 10 = 0$$

or

$$R_A + R_B = 10$$

Because the structure, the load, and the supports are all symmetrical about the centerline of the nutcracker, the two reactions labeled R_A and R_B must be equal. To understand why this is true, try the simple experiment illustrated below. Set up two scales a few inches apart, and lay a book across them. Ensure that the book is centered between the two scales. Then place a relatively heavy object like a full can of soup on top of the book. Gradually slide the soup can from one end of the book to the other, and watch the readings on the two scales as you move the can. You will notice that, when the can is perfectly centered between the two scales, the readings on the scales are exactly equal. At any other position, the readings are unequal. In this experiment, the soup can is the *load*; the book is the *structure*, and the scales directly measure the two *reactions*. The experiment clearly demonstrates that the reactions are equal if the loads, the supports, and the structure itself are symmetrical about a vertical centerline.



When the load is centered, the readings on the two scales are equal. Otherwise, the readings are unequal.

If the two reactions of our nutcracker truss are equal, then

$$R_A = R_B$$

If we substitute this expression into the equilibrium equation on the previous page, we get

$$R_A + R_B = R_B + R_B = 2R_B = 10$$

$$R_B = +5N = \underline{\underline{5N}} \uparrow$$

And since $R_A = R_B$,

$$R_A = \underline{\underline{5N}} \uparrow$$

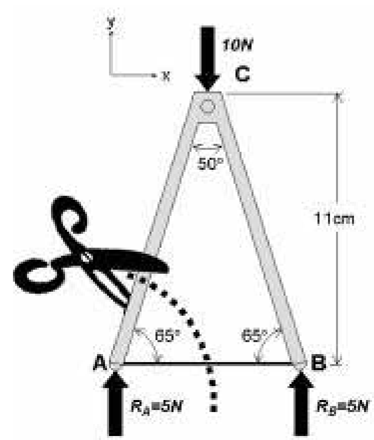
How do we calculate the reactions if the structure, the loads, or the supports are *not* symmetrical? This calculation requires the use of a third equilibrium condition—the condition that the sum of the "moments" about any point is zero. The concepts of "moments" and "moment equilibrium" are quite important but are beyond the scope of this book. All of the bridges we analyze and design here will be symmetrical.

Calculating Internal Member Forces

Once the reactions have been calculated, we can use a technique called the Method of Joints to calculate the internal member forces in a truss. To use the Method of Joints, we will use the following procedure:

1) Isolate one joint from the truss.
2) Draw a free body diagram of the joint.
3) Write and solve the equations of equilibrium to determine the member forces.
4) Repeat the process for the remaining joints.

Let's use our nutcracker truss to illustrate how the Method of Joints is used to analyze a structure.

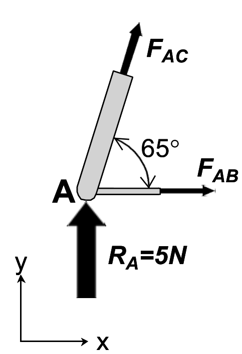


Step 1: Isolate one joint from the truss.

We'll begin our analysis with Joint A, at the lower left-hand corner of the truss. When you isolate a joint, imagine that you are physically cutting it out of the truss with a sharp scissors. You must cut through *all* of the members that connect the joint to the remainder of the structure. To isolate Joint A, you'll need to cut through Members AC (the handle) and AB (the string), as shown at right.

Step 2: Draw a free body diagram of the joint.

Now that we have cut Joint A out of the truss, we will draw a free body diagram of the joint itself. Like the diagram of the entire truss, this free body diagram must include any loads and reactions acting on the "body." Thus the upward reaction R_A is included, along with its known magnitude of 5 newtons.



In addition to the *external* loads and reactions, the free body diagram of Joint A must also include the *internal* member forces F_{AB} and F_{AC}. When we isolated the joint, we cut through Members AC and AB, thus "exposing" the internal forces in these members. We don't know the magnitudes of the two internal forces, so we simply label them with the variables F_{AB} and F_{AC}. We also don't know the directions of these forces. For now, we will simply assume that they are in *tension*. When a member is in tension, it pulls on the joint; thus, we indicate tension by showing the F_{AB} and F_{AC} vectors pointing *away from the joint,* along the centerlines of their respective members. Remember that we have only *assumed* F_{AB} and F_{AC} to be in tension. We will check this assumption when we solve the equations of equilibrium in Step 3.

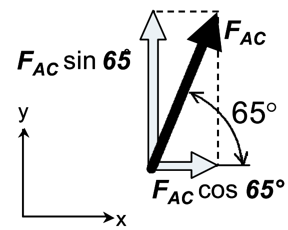
Step 3: Write and solve the equations of equilibrium.

When a structure is in equilibrium, every part of that structure must also be in equilibrium. We know that our nutcracker truss is in equilibrium, because it isn't moving; therefore, Joint A must be in equilibrium as well. Because Joint A is in equilibrium, we can write its two equilibrium equations. Let's start with the sum of forces acting in the y-direction. To write this equation, look at the free body diagram of Joint A, and identify every force that acts in the y-direction or has a component in the y-direction. Each of these forces must appear in the equilibrium equation. Assuming that the upward direction is positive,

$$\sum F_y = 0$$

$$+5 + F_{AC}\sin 65^\circ = 0$$

To write this equation, it was necessary to break the force vector F_{AC} into its x-component and y-component, as shown at right. The y-component is $F_{AC}\sin 65^\circ$, and because it points upward, it is positive. The x-component is $F_{AC}\cos 65^\circ$, but this component does not appear in the ΣF_y equilibrium equation, because it does not act in the y-direction.



Since this equation has only one unknown variable, we can calculate $\sin 65^\circ$ and solve for F_{AC} directly:

$$+5 + F_{AC}(0.906) = 0$$

$$F_{AC}(0.906) = -5$$

$$F_{AC} = \frac{-5}{0.906} = -5.52N$$

Because the answer is negative, our initial assumption about the direction of F_{AC} must have been incorrect. We assumed that the force F_{AC} is in tension; the negative answer tells us it is in compression. We can now write the final answer as

$$F_{AC} = \underline{\underline{5.52N\ (compression)}}$$

Note that, for internal forces, we do not show the direction of the force vector with an arrow; rather we simply label the force as either tension or compression.

Now we can write the second equilibrium equation—the sum of the forces in the x-direction—for Joint A. Again look at the free body diagram of the joint, and identify every force that acts in the x-direction or has a component in the x-direction. Include each of these forces in the equilibrium equation:

$$\sum F_x = 0$$

$$+F_{AB} + F_{AC}\cos 65^\circ = 0$$

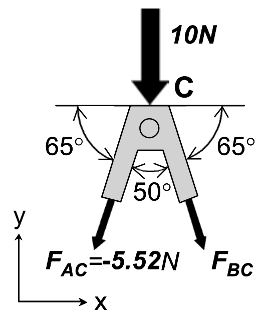
Note that the x-component of the force F_{AC} appears in this equation, while the y-component does not. To solve the equation, we must calculate $\cos 65^\circ$, substitute the value of F_{AC} we calculated above, and then solve for F_{AB}.

$$+F_{AB} + (-5.52)(0.423) = 0$$

$$F_{AB} = +2.33N = \underline{\underline{2.33N\ (tension)}}$$

Step 4: Repeat the process for the remaining joints.

Next we will apply the same solution process to Joint C. We begin by isolating the joint—cutting through the two handles, AC and BC. When we cut through these two members, we expose their internal forces F_{AC} and F_{BC}. Thus these two forces must be included on the free body diagram, along with the 10-newton load.



Note that both internal forces are again shown pointing away from the joint, indicating tension. This might appear to be incorrect, since we already know that F_{AC} is in compression. In fact, it is not an error but a technique to help prevent errors. To ensure that there is mathematical consistency when we move from joint to joint, it is best to *always* show internal member forces acting in tension. When our calculations show that a force is actually in compression, we write its magnitude as a negative number. The minus sign ensures that it is mathematically represented as a compression force in the equilibrium calculation. (We'll see how this works shortly.)

On the free body diagram, we can see that the two handles are connected together at an angle of 50°. It is also important to recognize that each handle forms an angle of 65° measured from horizontal. If you have studied plane geometry, you should be able to prove that this is true.

Again, once we have carefully drawn the free body diagram of the joint, we can write an equilibrium equation to determine the unknown member force F_{BC}. In this case, either equation will do the job. Let's use the sum of the forces in the y-direction:

$$\sum F_y = 0$$

$$-10 - F_{AC}\sin 65^\circ - F_{BC}\sin 65^\circ = 0$$

Now we can substitute the calculated value of F_{AC} and solve for F_{BC}. Remember that F_{AC} is in compression, so the value you substitute must have a minus sign.

$$-10 - (-5.52)(0.906) - F_{BC}(0.906) = 0$$

$$-10 + 5 - F_{BC}(0.906) = 0$$

$$-F_{BC}(0.906) = 5$$

$$F_{BC} = \frac{-5}{0.906} = -5.52N = \underline{\underline{5.52N\ (compression)}}$$

We shouldn't be surprised that F_{BC} turns out to be exactly the same as F_{AC}. Given that the structure, loads, and reactions are all symmetrical, it certainly makes sense that the compression forces in the two handles are also the same. Nonetheless, it is reassuring that our mathematical equations and our common sense both produce the same result. For further reassurance, try writing the second equilibrium equation, $\Sigma F_x = 0$, for Joint C. This calculation will also show that $F_{BC} = -5.52$ newtons. If you write the equilibrium equations for Joint B, you will again find that $F_{BC} = 5.52$ newtons (compression) and $F_{AB} = 2.33$ newtons (tension).

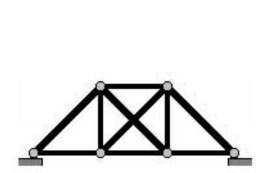
Static Determinacy and Stability

The Method of Joints is a simple, powerful tool for calculating the forces in truss members. Unfortunately, the method does not work for all trusses. If a structure has more unknown member forces and reactions than the number of available equilibrium equations, then the Method of Joints is not sufficient to perform the structural analysis. A structure that cannot be analyzed using the equations of equilibrium alone is called **statically indeterminate**. A structure that *can* be analyzed using the equations of equilibrium alone is called **statically determinate**. Only statically determinate trusses can be analyzed with the Method of Joints.

A statically determinate truss with two reactions must satisfy the mathematical equation

$$2j = m + 3$$

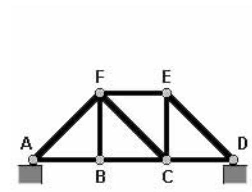
where ***j*** is the number of joints and ***m*** is the number of members. For example, our nutcracker truss has 3 members and 3 joints. Substituting these numbers into the equation above, we find that ***2j*** and ***m+3*** are both equal to 6, so the mathematical condition for static determinacy is satisfied. If ***2j is less than m+3***, then the truss is *statically indeterminate*. For example, the truss at right has 6 joints and 10 members. Thus ***2j*** is 12, and ***m+3*** is 13. Since ***2j*** is less than ***m+3***, the structure is indeterminate. Such a structure cannot be analyzed using the equations of equilibrium alone. If you tried to use the Method of Joints to analyze this truss, you would find that you have more unknown forces than you have equations available to solve for them. It is possible to analyze a statically indeterminate structure, but the solution process requires advanced engineering concepts that are beyond the scope of this book.



A statically indeterminate truss.

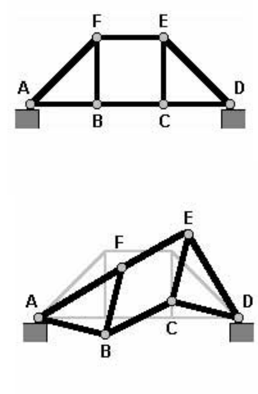
If ***2j is greater than m+3***, then the truss is **unstable**. An unstable truss does not have enough members to form a rigid framework. Such a structure cannot carry any load.

In general, a truss is stable if all of its members are arranged in a network of interconnected triangles. For example, the simple truss at right is composed of 6 joints and 9 members, which together form four interconnected triangles (ABF, BCF, CEF, and CDE). This truss also satisfies the mathematical condition for determinacy, since both ***2j*** and ***m+3*** are equal to 12.



A statically determinate truss.

If member CF is removed, however, the truss becomes unstable. Without its diagonal member, the center panel of the truss now consists of a rectangle (BCEF) formed by four members, rather than two triangles (BCF and CEF). This configuration is unstable because there is nothing to prevent the rectangle BCEF from distorting into a parallelogram, as shown below. (Remember that we assume all truss joints to be frictionless pins.) For this truss, ***2j*** is still 12, while ***m+3*** is only 11. Since *2j* is greater than *m+3*, the mathematical test confirms our observation that the truss is unstable.



An unstable truss.

As you might expect from this example, an unstable truss can generally be made stable by simply adding members until an appropriate arrangement of interconnected triangles is achieved.

Factor of Safety

When an engineer designs a structure, he or she must consider many different forms of *uncertainty*. There are three major types of uncertainty that affect a structural design:

- There is always substantial uncertainty in predicting the loads a structure might experience at some time in the future. Wind, snow, and earthquake loads are highly unpredictable. The engineer can never be certain of the maximum number of people that might occupy an apartment building or the weight of the heaviest truck that might cross a bridge. Truck weights are regulated by law in the United States, but illegally heavy trucks occasionally do drive our highways, and it only takes one of them to collapse a bridge. You can post a 20-ton Load Limit sign on a bridge, but that doesn't mean the driver of a 30-ton truck won't try to cross it anyway.

The loads applied to structures are highly unpredictable.

- The strengths of the materials that are used to build actual bridges are also uncertain. Manufacturers of construction materials generally pay careful attention to the quality of their products; nonetheless, it is always possible for a batch of substandard steel or concrete to be used in a structure. Even the most conscientious construction contractors occasionally make mistakes on a project, and some construction errors can reduce the ability of a structure to carry load.
- The mathematical models we use for structural analysis and design are never 100% accurate. We have already seen this in our discussion of structural models—actual trusses do not have perfectly straight members or frictionless pinned connections. Yet we must make these sorts of simplifying assumptions, or the truss analysis simply cannot be performed.

The engineer accounts for all forms of uncertainty by making the structure somewhat stronger than it really needs to be—by using a **factor of safety** in all analysis and design calculations. In general, when it is used in the analysis of an existing structure, the factor of safety is a defined as

$$\textit{Factor of Safety} = \frac{\textit{Failure Level}}{\textit{Actual Level}}$$

In a truss, the *actual* force in a member is called the *internal member force*, and the force at which *failure* occurs is called the *strength*. Thus we can rewrite the definition of the factor of safety as

$$\textit{Factor of Safety} = \frac{\textit{Strength}}{\textit{Internal Member Force}}$$

For example, if a structural member has an internal force of 5000 pounds and a strength of 7500 pounds, then its factor of safety, ***FS***, is

$$\boldsymbol{FS = \frac{7500}{5000} = 1.5}$$

If the factor of safety is less than 1, then the member or structure is clearly unsafe and will probably fail. If the factor of safety is 1 or only slightly greater than 1, then the member or structure is nominally safe but has very little margin for error—for variability in loads, unanticipated low member strengths, or inaccurate analysis results. Most structural design codes specify a factor of safety of 1.6 or larger (sometimes considerably larger) for structural members and connections.

In Learning Activity #5, we'll see how the factor of safety is applied in the design process.

On an Actual Bridge Project

Load and Resistance Factor Design

The factor of safety has been used in structural engineering for over a century. In recent years, however, a new design philosophy called load and resistance factor design (LRFD) has become increasing popular. LRFD is based on the idea that the largest loading a structural member experiences in its lifetime must be less than the smallest possible strength of that member. In an LRFD-based design, the engineer estimates this "largest loading" by adjusting the loads used in the structural analysis. All loads are multiplied by a code-specified load factor—a number that is always greater than 1. The actual magnitude of the load factor depends on how uncertain the loads are. The self-weight of a structure can be predicted accurately, so it has a relatively low load factor (normally 1.2 to 1.4). Wind, traffic, and earthquake loads are much more unpredictable, so their load factors are usually much higher. To estimate the smallest possible strength of a member, the engineer multiplies the nominal member strength by a code-specified resistance factor—a number that is always less than 1. The resistance factor accounts for the possibility of understrength materials, fabrication errors, and other uncertainties that may cause a member to be weaker than the engineer intended. Ultimately load factors and resistance factors serve the same function as the factor of safety—they ensure the safety of a structure by providing a margin for error. Many experts view the LRFD as a superior design philosophy, because it more accurately represents the sources of uncertainty in structural design.

The Learning Activity

The Problem

The Need

One year after the completion of the new Grant Road Bridge, the Hauptville Town Engineer inspects the structure and finds that it is performing well. Though the bridge has been carrying a lot of traffic, its structural members show no signs of distress or deterioration. Nonetheless, the Town Engineer is still somewhat concerned about the bridge. Because of a major construction project nearby, many heavily loaded dump trucks have been using Grant Road recently. What if one of these trucks is heavier than the legal weight limit? How much of an overload would cause the structure to collapse? The Town Engineer decides to perform a complete structural evaluation to determine the overall level of safety of the Grant Road Bridge. He begins by hiring Universal Structural Materials Assessment, Inc. to test the strength of the structural members used in the bridge. (We did this part of the structural evaluation in Learning Activity #2.) Once the Engineer has received the test results from Universal, he is ready to begin his analysis.

Your Job

You are the Town Engineer of Hauptville. Your job is to analyze the Grant Road Bridge and evaluate its overall level of safety. Specifically, you must calculate the factor of safety for every member in one of the main trusses, then determine the overall safety factor for the structure.

As the Town Engineer, you have the professional responsibility to protect the health and safety of the people who use this bridge. You fulfill this responsibility by performing the structural evaluation conscientiously—by using good judgment, by performing calculations carefully and accurately, and by asking a colleague to check your work.

The Solution

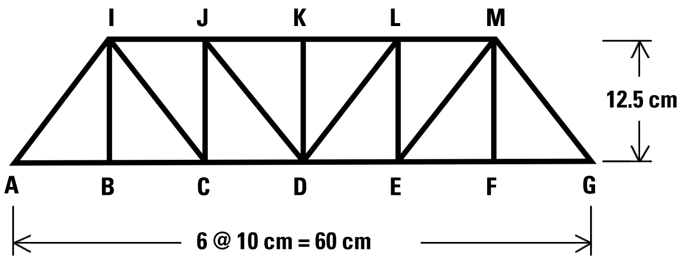
The Plan

Our plan to conduct the structural analysis and evaluation of the Grant Road Bridge consists of the following tasks:

- Create the structural model.
- Check the structural model for static determinacy and stability.
- Calculate the reactions.
- Calculate the internal member forces.
- Determine the strengths of the members
- Calculate the factor of safety for every member in the structure
- Evaluate the safety of the structure.
- Check our assumptions.

Create the Structural Model

To model the Grant Road Bridge, we must define (1) the geometry of the structure, (2) the loads, and (3) the supports and reactions. We begin by idealizing the three-dimensional bridge structure as a pair of two-dimensional Pratt trusses. Since these two trusses are identical, we only need to analyze one of them. The geometry of the truss is shown below. The dimensions indicate the locations of the *centerlines* of the members. Joints are identified with letters—the same letter designations that were used on the bridge plans provided in Learning Activity #1. To facilitate the analysis, we will assume that the truss members are perfectly straight, the joints are frictionless pins, and the loads are applied only at the joints. We will also assume that the weight of the truss itself is zero.

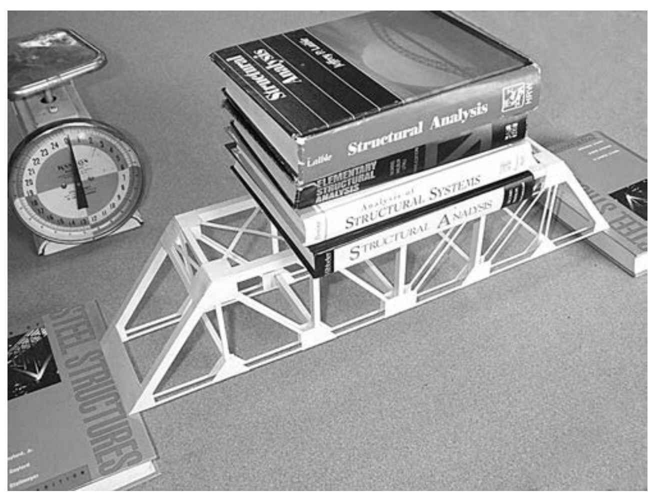


Q1

Why did we assume that the weight of the truss is zero?

Obviously, the actual weight of the truss is not zero. Why did we make this assumption, when we know it is not true? How do you think it will affect the accuracy of our structural analysis?

When we load-tested the Grant Road Bridge in Learning Activity #1, we applied the load in two different ways—with a stack of books placed on the top chord and with a bucket of sand suspended from the floor beams. Before we can define the loads for our structural model, we need to decide which of these two loading configurations to use. As a general rule, a structural evaluation should be based on the most severe loading condition—the one that produces the highest member forces. If the analysis shows that the truss is safe for the most severe loading, then the structure will certainly be safe for less severe ones. Unfortunately, in this case, it is not immediately obvious which of the two loading configurations is more severe. The best we can do is to make an assumption and check it later. For now, we will assume that the top-chord loading, shown here, is more severe.



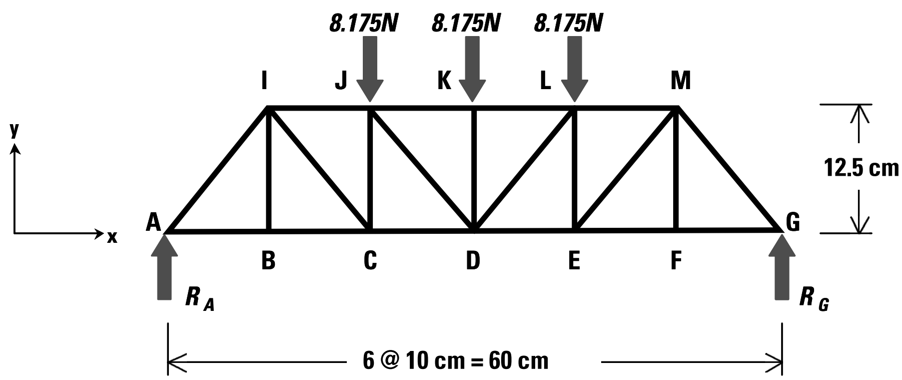
We will assume that the top chord loading is more severe, then check the bottom-chord loading later.

Having decided on the location of the load, we must now determine its magnitude. In Learning Activity #2, we applied the equation ***W=mg*** to determine that the weight of a 5-kilogram mass is 49.05 newtons. When we placed the stack of books onto the top chord of the truss, the weight of the stack was supported on joints J, J′, K, K′, L, and L′. We can reasonably assume that the weight of the books is distributed equally to these six joints. Therefore, the downward force applied to each joint is

$$Load\ per\ Joint = \frac{Total\ Load}{Number\ of\ Joints} = \frac{\mathbf{49.05N}}{\mathbf{6}} = \mathbf{8.175}N$$

Since there are two main trusses, three ***8.175N*** loads will be applied to each truss. Since all of the loads are downward, and the bridge is supported only at its ends, we will add upward reactions R_A and R_G at joints A and G.

A complete free body diagram of the truss looks like this:



Check Static Determinacy and Stability

Before we can use the equations of equilibrium to analyze this truss, we must first verify that it is statically determinate and stable. The mathematical condition for static determinacy and stability is

$$2j = m + 3$$

where ***j*** is the number of joints and ***m*** is the number of members. Our truss from the Grant Road Bridge has 12 joints and 21 members. Substituting these numbers into the equation above, we find that ***2j*** and ***m+3*** are both equal to 24, so the mathematical condition for static determinacy and stability is satisfied. Furthermore, we note that the truss is composed entirely of interconnected triangles, which confirms our conclusion that the structure is stable.

Calculate Reactions

On the free body diagram above, the forces R_A and R_G are the unknown reactions at Joints A and G. We know that the truss is in equilibrium; therefore, the sum of all forces acting on the structure must be zero. Since all of the forces—loads and reactions—are acting in the y-direction, only one of our two equilibrium equations is relevant to the calculation of reactions:

$$\sum F_y = 0$$

$$R_A + R_G - 8.175 - 8.175 - 8.175 = 0$$

LEARNING ACTIVITY #3

Since the structure, the loads, and the reactions are all symmetrical about the centerline of the truss, the two reactions R_A and R_G must be equal. (The centerline of the truss is a vertical line passing through Member DK). Substituting $R_A = R_G$ into the equilibrium equation above, we get

$$R_A + R_A - 24.525 = 0$$

$$2R_A = 24.525$$

$$R_A = 12.26N \uparrow$$

And since $R_A = R_G$, then

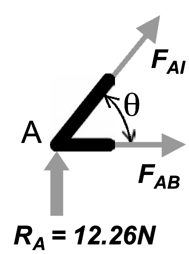
$$R_G = 12.26N \uparrow$$

Calculate Internal Member Forces

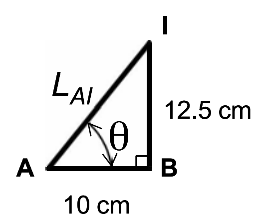
We will use the Method of Joints to calculate the internal force in each member of the truss. To apply this method, we will isolate a joint from of the structure, cutting through the attached members and exposing their internal member forces. We will draw a free body diagram of the joint, then use the equations of equilibrium to determine the unknown member forces. We will repeat the process for successive joints, until we have calculated all of the internal member forces in the structure.

Joint A

We'll start by isolating Joint A and drawing a free body diagram of it. The free body diagram must show *all* forces acting on the joint. Thus the reaction R_A is shown, along with its known magnitude of 12.26N. The member forces F_{AI} and F_{AB} are also included on the diagram. Because we do not know the magnitudes or directions of these forces, we simply show them in variable form, and we assume their directions to be in tension. To indicate that a member force is in tension, we draw the force vector pointing away from the joint, along the centerline of the member.



Before we can write the equilibrium equations for this joint, we need to figure out what the angle θ is. Actually, we don't really need to know the angle itself; rather, we only really need to know the sine and cosine of the angle—**sin**θ and **cos**θ. We can determine the sine and cosine directly from the geometry of the truss. Note that Members AB, AI, and BI form a right triangle, with Member AI as the hypotenuse. We can apply the Pythagorean Theorem to calculate the length, L_{AI}, as follows:



$$L_{AI} = \sqrt{10^2 + 12.5^2} = 16.01cm$$

Now we can apply the basic definitions of the sine and cosine to find **sin**θ and **cos**θ*:*

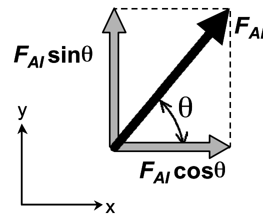
$$\sin\theta = \frac{opposite}{hypotenuse} = \frac{12.5}{16.01} = 0.7809$$

$$\cos\theta = \frac{adjacent}{hypotenuse} = \frac{10}{16.01} = 0.6247$$

We are finally ready to write the equilibrium equations for Joint A. We will start with the equation for the sum of forces in the y-direction. Assuming that the upward direction is positive,

$$\sum F_y = 0$$

$$12.26 + F_{AI} \sin\theta = 0$$



To write this equation, we had to represent the force F_{AI} in terms of its x-component and y-component. The y-component is F_{AI}**sin**θ, and its direction is upward, so it is positive in the equilibrium equation. The x-component is F_{AI}**cos**θ; however, this component is not included in the ΣF_y equilibrium equation, because it does not act in the y-direction.

We can now substitute the known value of **sin**θ into the equilibrium equation, and solve for the unknown force F_{AI}.

$$12.26 + F_{AI}(0.7809) = 0$$

$$F_{AI}(0.7809) = -12.26$$

$$F_{AI} = -\frac{12.26}{0.7809} = -15.70N$$

Because the answer is negative, our initial assumption about the direction of F_{AI} was incorrect. We assumed that F_{AI} is in tension. The negative member force indicates that it is in compression. Thus our final answer is

$$F_{AI} = \underline{\underline{15.70N \text{ (compression)}}}$$

Now we can write the equilibrium equation for forces in the x-direction. Assuming that the positive direction is to the right,

$$\sum F_x = 0$$

$$F_{AB} + F_{AI} \cos\theta = 0$$

We know **cos**θ, and we have just solved for F_{AI}. We can substitute these values into the equilibrium equation and solve for F_{AB}. But be careful! When you substitute F_{AI}, don't forget the minus sign.

$$F_{AB} + (-15.70)(0.6247) = 0$$

$$F_{AB} = +9.81N$$

Because the answer is positive, our assumption about the direction of F_{AB} was correct.
The final answer is

$$F_{AB} = \underline{\underline{9.81N \text{ (tension)}}}$$

Q2

Why did we choose to do Joint A first?

Is there a reason why Joint A was a good place to start this analysis? What would have happened if we had started with a different joint?

Q3

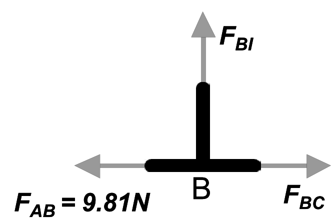
Why did we choose to solve the y-direction equilibrium equation first?

Is there a reason why it was a good idea to solve $\Sigma F_y = 0$ before solving $\Sigma F_x = 0$?

Joint B

At this point, we should analyze Joint B. It has only three connected members, and we already know the internal force in one of the three (Member AB). Thus there are only two unknown forces, and we will be able to solve for them with the two available equilibrium equations.

Again we draw a free body diagram of the joint, with all member forces assumed to be in tension—pointing away from the joint. The known magnitude of the force F_{AB} is included on the diagram. The equilibrium equation for forces in the x-direction is



$$\sum F_x = 0$$

$$-9.81 + F_{BC} = 0$$

$$F_{BC} = +9.81N = \underline{\underline{9.81N\ (tension)}}$$

The equilibrium equation for forces in the y-direction produces an interesting result:

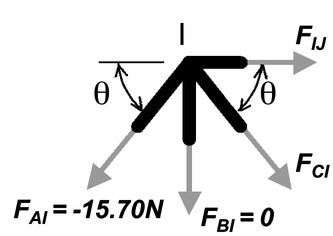
$$\sum F_y = 0$$

$$F_{BI} = 0$$

It should come as no surprise that this member has zero internal force. When you load-tested the Grant Road Bridge in Learning Activity #1, you should have noticed that this member—the hip vertical—was slack. It appeared to have no internal force at all. Now we have verified our observation using the Method of Joints!

Joint I

It takes some careful thought to recognize that Joint I should be the next joint we analyze. As the free body diagram indicates, this joint has four connected members and, therefore, it also has four internal member forces. Note, however, that we have already calculated two of these—F_{AI} and F_{BI}. Thus there are only two unknown forces, which we can calculate with our two equilibrium equations.



Note that all four of the force vectors are pointing away from the joint, even though we already know that one of them, F_{AI}, is in compression. The negative magnitude of F_{AI} ensures that it is mathematically represented as a compression force.

It is important to note that both angles labeled as θ on the free body diagram are exactly the same as the angle θ on the diagram of Joint A. (If you can't see that these angles are all equal, prove it to yourself by drawing the corresponding right triangles, just as we did for Joint A.) Thus the values we calculated for **sin**θ and **cos**θ for Joint A are still valid here.

If we begin with the equilibrium equation in the y-direction, we will be able to solve for F_{CI} directly:

$$\sum F_y = 0$$

$$-F_{AI}\sin\theta - F_{BI} - F_{CI}\sin\theta = 0$$

$$-(-15.70)(0.7809) - 0 - F_{CI}(0.7809) = 0$$

$$F_{CI} = +15.70 = \underline{\underline{15.70N\ (tension)}}$$

Now we can use the second equilibrium equation to solve for F_{IJ}:

$$\sum F_x = 0$$

$$-F_{AI}\cos\theta + F_{CI}\cos\theta + F_{IJ} = 0$$

$$-(-15.70)(0.6247) + 15.70(0.6247) + F_{IJ} = 0$$

$$F_{IJ} = -19.62N = \underline{\underline{19.62N\ (compression)}}$$

Joint C

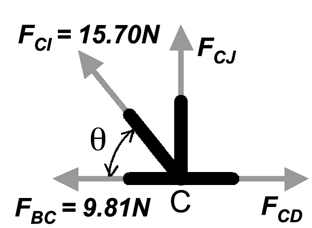
Next we will calculate the unknown member forces at Joint C.

Q4

Why Joint C?

Why was Joint C the best joint to analyze at this point in the solution process?

Based on the free body diagram of the joint, we can write the two equilibrium equations and solve for the two unknown member forces as follows:



$$\sum F_x = 0$$

$$-F_{BC} - F_{CI}\cos\theta + F_{CD} = 0$$

$$-9.81 - (15.70)(0.6247) + F_{CD} = 0$$

$$F_{CD} = +19.62N = \underline{\underline{19.62N \text{ (tension)}}}$$

$$\sum F_y = 0$$

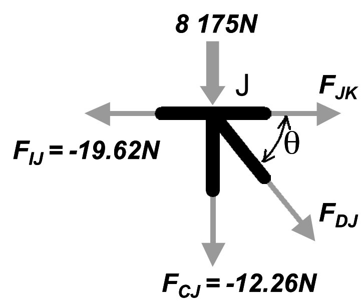
$$F_{CI}\sin\theta + F_{CJ} = 0$$

$$(15.70)(0.7809) + F_{CJ} = 0$$

$$F_{CJ} = -12.26N = \underline{\underline{12.26N \text{ (compression)}}}$$

Joint J

The free body diagram of Joint J is shown at right. Note that the 8.175N load at Joint J *must* be included on the diagram. (Failure to put loads on the free body diagram is one of the most common errors in truss analysis.) We can write the two equilibrium equations and solve for the two unknown member forces as follows:



$$\sum F_y = 0$$

$$-8.175 - F_{CJ} - F_{DJ}\sin\theta = 0$$

$$-8.175 - (-12.26) - F_{DJ}(0.7809) = 0$$

$$F_{DJ}(0.7809) = 4.085$$

$$F_{DJ} = +5.23N = \underline{\underline{5.23N \text{ (tension)}}}$$

$$\sum F_x = 0$$

$$-F_{IJ} + F_{DJ}\cos\theta + F_{JK} = 0$$

$$-(-19.62) + (5.23)(0.6247) + F_{JK} = 0$$

$$F_{JK} = -22.89N = \underline{\underline{22.89N \text{ (compression)}}}$$

Q5

Can you apply the Method of Joints to calculate a member force?

Which joint should you analyze to determine the member force F_{DK}? Solve the appropriate equilibrium equations to show that $F_{DK}=8.175$N *(compression)*.

Summary of Structural Analysis Results

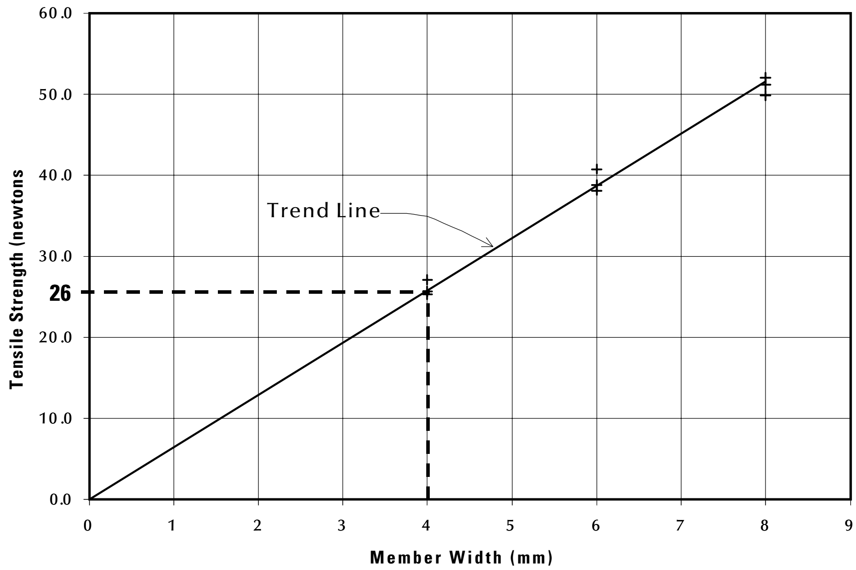
At this point, we have only analyzed half of the truss. However, if we take advantage of symmetry, we can determine the internal forces in all remaining members without doing any further calculations. When we determined the reactions R_A and R_G, we noted that these two forces must be equal because the structure, its loads, and its reactions are all symmetrical about the centerline of the truss. The same principle holds true for internal member forces. Because the structure, loads, and reactions are all symmetrical, the member forces must also be symmetrical about the centerline. Members that are "mirror images" of each other have equal internal forces. F_{GM} and F_{AI} must be equal; F_{LM} and F_{IJ} must be equal; F_{DJ} and F_{DL} must be equal, and so forth.

So we're done! The results of the analysis are summarized in the table below.

Members	Force		Members	Force	
AB, FG	9.81 N	(tension)	BI, FM	0 N	
BC, EF	9.81 N	(tension)	CI, EM	15.70 N	(tension)
CD, DE	19.62 N	(tension)	CJ, EL	12.26 N	(compression)
IJ, LM	19.62 N	compression)	DJ, DL	5.23 N	(tension)
JK, KL	22.89 N	(compression)	DK	8.175 N	(compression)
AI, GM	15.70 N	(compression)			

Determine the Strengths of the Members

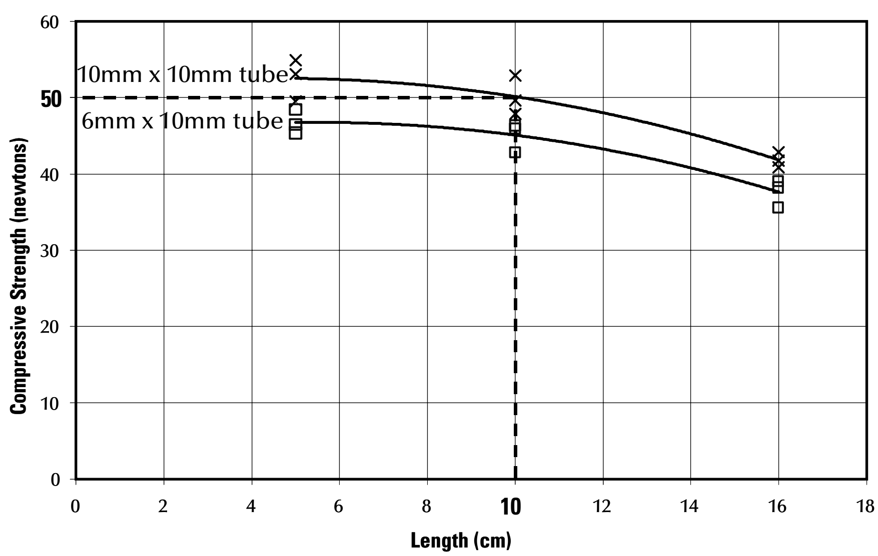
Now that we have calculated the force in each member, we must determine the corresponding strength of each member. To do this, we will use the graphs we developed in Learning Activity #2. We'll start with the *bars*—the bottom chords, diagonals, and hip verticals. The table above tells us what we have already observed in our Grant Road Bridge model—that all of the bars are in tension (except the hip verticals, which have zero internal force). Thus we need to determine the *tensile strength* of the bars. All of the bars used in the Grant Road Bridge are 4mm wide. Using the tensile strength vs. member width graph we developed in Learning Activity #2, we find the tensile strength of a 4mm bar to be 26 newtons, as shown below.



Determining the tensile strength of a 4mm bar

Note, however, that all of the bottom chords, diagonals, and hip verticals are actually *doubled* 4mm bars. Thus the tensile strength of these members is exactly twice that of a single 4mm bar, or 52 newtons.

Our structural analysis shows that all of the tubes—the top chords, the end posts, and the interior verticals—are in compression. Thus we must determine the compressive strength of these members, using the strength vs. length graph we developed in Learning Activity #2. The top chord members are all 10mm x 10mm tubes, and each has a length of 10 centimeters. The strength of these members is approximately 50 newtons, as indicated below.



Determining the compressive strength of a 10mm x 10mm tube that is 10 centimeters long

Q6

Can you determine the strength of a member?

What is the compressive strength of the vertical tube members (CJ, DK, and EL) and the end posts (AI and GM)?

Calculate the Factor of Safety

Once we know the strength of a member and the internal force it is actually experiencing, we can calculate its factor of safety. For the bottom chord member CD, the factor of safety is:

$$FS_{CD} = \frac{Strength}{Internal\ Member\ Force} = \frac{52}{19.62} = 2.7$$

For the top chord member JK, the factor of safety is

$$FS_{JK} = \frac{Strength}{Internal\ Member\ Force} = \frac{50}{22.89} = 2.2$$

Q7

Can you calculate the factor of safety for a member?

Calculate the factor of safety for all remaining members in the truss, and add them to the summary table below (along with the member strengths not already recorded in the table).

Members	Force		Strength	FS
AB, FG	9.81 N	(tension)	52	
BC, EF	9.81 N	(tension)	52	
CD, DE	19.62 N	(tension)	52	2.7
IJ, LM	19.62 N	(compression)	50	
JK, KL	22.89 N	(compression)	50	2.2
AI, GM	15.70 N	(compression)		
BI, FM	0 N	--	52	--
CI, EM	15.70 N	(tension)	52	
CJ, EL	12.26 N	(compression)		
DJ, DL	5.23 N	(tension)	52	
DK	8.175 N	(compression)		

Evaluate the Structure

As the Town Engineer of Hauptville, you have finished what you set out to do—a complete structural evaluation of the main trusses of the Grant Road Bridge. Yet the results of these calculations are just numbers. They are of little use, until you study them, think critically about them, and draw meaningful conclusions from them.

Once you have completed the summary table on the previous page, you should be able to make the following observations:

- Members JK and KL have a factor of safety of 2.2—the smallest of any member in the truss. Since the failure of Member JK or Member KL would cause the entire structure to collapse, we can say that the factor of safety of the *entire structure* is 2.2.
- Since 2.2 is obviously larger than 1, our analysis tells us that the structure will not collapse when the 5 kg mass is placed on the top chord. Because the factor of safety is considerably larger than 1, we can have a high degree of confidence that the structure will not fail, even if we made some minor errors in construction or if the actual load is significantly larger than 5 kg.
- Theoretically, the bridge would collapse if the mass of the stack of books at mid-span were increased to ***(5 kg)(2.2)=11.0 kg***.
- Many members of the truss have safety factors that are substantially larger than 2.2. These members are actually much stronger than they need to be.

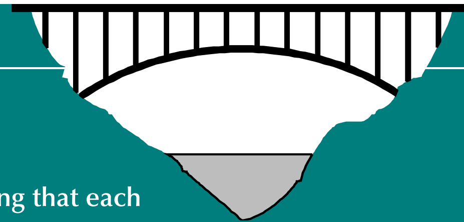
Q8

Why are some truss members stronger than they need to be?

For example, Member CI is a doubled 4mm bar with a safety factor of 3.3. Had the structural engineer chosen to use a doubled 3mm bar for this member, the safety factor would still be 2.5. The member would use less material; and because its safety factor would still be greater than 2.2, the overall safety of the structure would not be adversely affected. Why did the structural engineer choose not to use a smaller member size?

It is important to note that this structural evaluation is valid only for one particular loading. If we change either the magnitude or the position of the load, the member forces and factors of safety will also change.

On an Actual Bridge Project

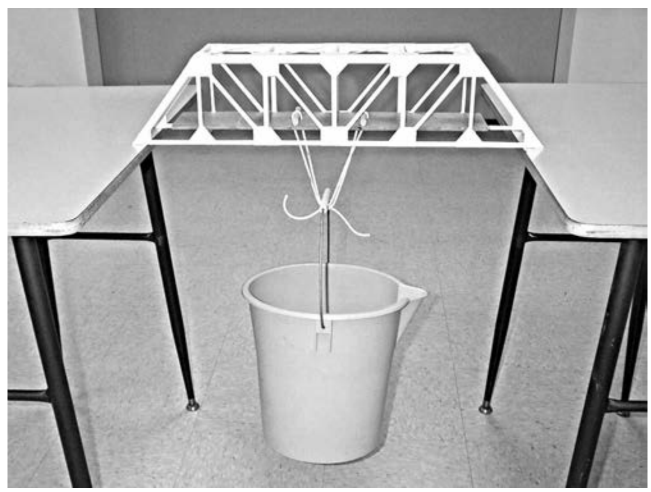


Deflections are important too.

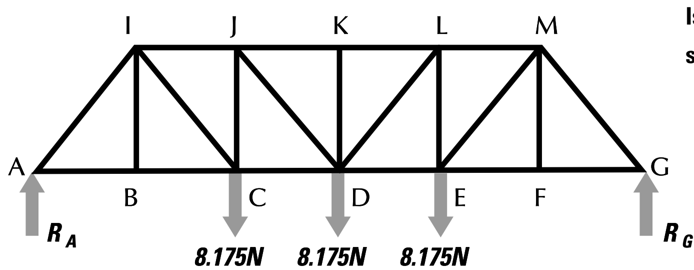
We evaluated the Grant Road Bridge by checking that each member in the structure can carry load safely. On an actual bridge project, the engineer would also check to ensure that the *deflections* are acceptably small. A **deflection** is the distance a structure moves when it is loaded. A bridge *always* deflects when it is loaded. On a well-designed bridge, these deflections are so small that they are imperceptible to drivers and pedestrians crossing the span. If a bridge is too flexible, then drivers and pedestrians will feel its movement and will perceive the structure to be unsafe—even if its strength entirely adequate. The engineer is responsible for ensuring not only that the bridge *is safe*, but also that the public *perceives it to be safe*. Thus the engineer carefully computes the structure's deflections under various loading conditions and ensures that these computed deflections comply with the appropriate design codes.

Bottom-Chord Loading

The analysis above was based on a number of assumptions. Perhaps the most important of these was our assumption that placing the load on the top chord of the truss is more severe than suspending the load from the floor beams. We now have the analytical tools to check this assumption. Since the floor beams are attached to the bottom chords, loading the floor beams is essentially the same as loading the bottom chord joints of the truss. Thus our revised structural model should have the three 8.175 N loads applied at the bottom chord, as shown below:



Is bottom-chord loading more or less severe than top-chord loading?



If we repeat the structural analysis using this new loading condition, we get the following results:

Members	Force		Members	Force	
AB, FG	9.81 N	(tension)	BI, FM	0 N	
BC, EF	9.81 N	(tension)	CI, EM	15.70 N	(tension)
CD, DE	19.62 N	(tension)	CJ, EL	4.09 N	(compression)
IJ, LM	19.62 N	compression)	DJ, DL	5.23 N	(tension)
JK, KL	22.89 N	(compression)	DK	0 N	(compression)
AI, GM	15.70 N	(compression)			

LEARNING ACTIVITY #3

Q9

Can you apply the Method of Joints to analyze a truss?

Use the Method of Joints to analyze the Pratt truss with bottom-chord loading. Prove that the results in the table above are correct.

Note that moving the loads from the top chord to the bottom chord caused the internal forces to change *only* in Members CJ, EL, and DK. In all three cases, the forces got smaller. The forces in the most heavily loaded members—the top and bottom chords and the end posts—remained unchanged; thus the overall factor of safety of the structure remains unchanged. We can conclude that using the top-chord loading in Learning Activity #1 (and in our initial structural analysis) was entirely appropriate. The bottom-chord loading is more realistic, because real bridges carry traffic loads on their floor beams; however, the top-chord loading is much easier to do and produces nearly identical structural analysis results.

Q10

Can members with zero force be removed?

In the analysis above, Members BI, FM, and DK have zero internal force. It would seem that the truss does not need these members to carry load, and we might simply remove them from the structure to save material. Do you think these zero-force members can safely be removed from the truss?

Q11

Can you analyze a different truss?

Select any truss from the Gallery of Structural Analysis Results (Appendix B), and calculate the internal forces in all of its members.

Conclusion

In this learning activity, we applied concepts from geometry, trigonometry, algebra, and physics to calculate the internal forces in every member of a truss. In doing so, you saw how math and science are applied to solve an important, real-world problem. You also saw how data from laboratory experiments—the strength tests we did in Learning Activity #2—can be integrated into the solution of an engineering problem. Most important, you learned to use an important analysis tool called the Method of Joints. This technique isn't easy! It requires you to construct and solve equilibrium equations—lots of them—while paying careful attention to the magnitudes and directions of forces. Wouldn't it be great if we could use the computer to do this work for us? Well, we can. In Learning Activity #4, we will use the West Point Bridge Designer software to analyze and evaluate a truss bridge.

Answers to the Questions

1) Why did we assume that the weight of the truss is zero? We *could* include the self-weight of the bridge in our structural analysis, but doing so would *greatly* complicate the analysis. In Learning Activity #1, we found that the mass of our Grant Road Bridge model is about 55 grams. This mass is very small in comparison with the 5 kilogram mass the bridge is designed to carry. Including the self-weight in our analysis would change our calculated member forces by only about one percent. For the sake of simplicity, we can ignore self-weight, and recognize that this assumption has a very small effect on the accuracy of the structural analysis.

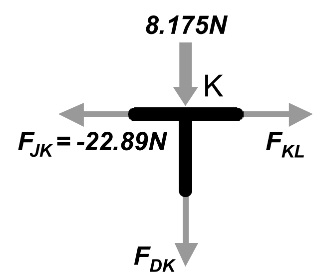
2) Why did we choose to do Joint A first? At any given joint, we can write two equilibrium equations—$\Sigma F_x=0$ and $\Sigma F_y=0$. With only two available equations, we can solve for only two unknown member forces at each joint. At the start of the solution process, Joint A has just two unknown member forces—F_{AB} and F_{AI}. Joint G is the only other joint in the entire truss with only two unknown forces. All of the others have three or more. Thus the solution process should start at either Joint A or Joint G.

Note that the reaction force R_A is also applied at Joint A. Had this force also been unknown, it would have been impossible to solve the equilibrium equations at this joint—there would have been three unknowns and only two equations. That's why it is generally necessary to solve for reactions before we use the Method of Joints to calculate member forces.

3) At Joint A, why did we choose to solve the y-direction equilibrium equation first? At Joint A, the x-direction equilibrium equation has two unknown member forces, F_{AB} and F_{AI}. Had we written this equation first, we would have been unable to solve for either of the unknowns immediately. The y-direction equation includes only one unknown force, F_{AI}. By solving $\Sigma F_y=0$ first, we were able to solve for F_{AI} directly. Then, when we wrote $\Sigma F_x=0$, we could substitute the known value of F_{AI} and solve for F_{AB}. At any given joint, we can often (but not always) avoid the chore of solving two equations simultaneously by identifying an equilibrium equation that has only one unknown member force—and solving it *first*.

4) After solving for the unknown member forces at Joints A, B, and I, why did we choose Joint C? At Joints A, B, and I we calculated the member forces F_{AB}, F_{AI}, F_{BC}, F_{BI}, F_{CI}, and F_{IJ}. At this point in the solution process, we already knew two of the four member forces at Joint C—F_{BC} and F_{CI}. Only F_{CD} and F_{CJ} were unknown. So we selected Joint C because it had only two unknown member forces, which could be solved with our two equations of equilibrium. As a general rule, when using the Method of Joints to analyze a truss, always select a joint with only two unknown member forces as the next step in the analysis.

5) Can you apply the Method of Joints to calculate the force in Member DK? The best joint to analyze in order to determine the member force F_{DK} is Joint K. The free body diagram of this joint is shown at right. From this diagram, we can calculate F_{DK} directly from the y-direction equilibrium equation



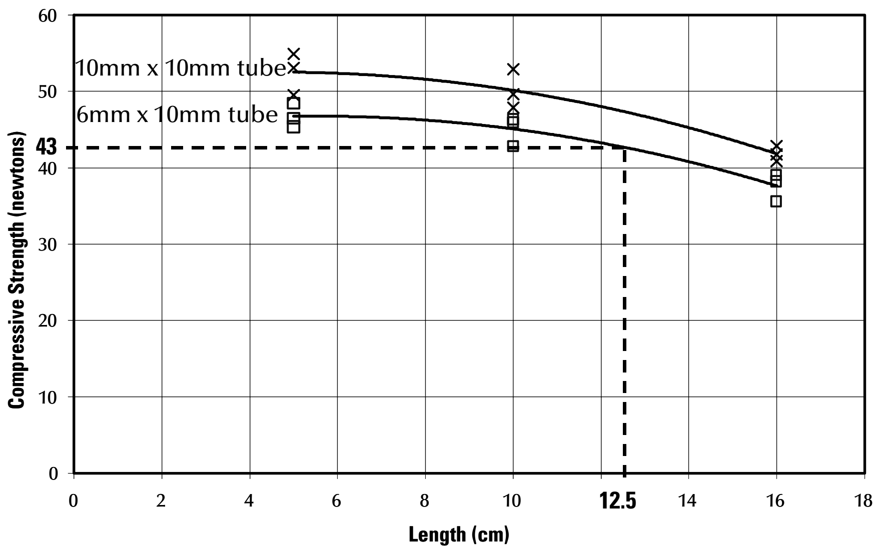
$$\sum F_y = 0$$

$$-8.175 - F_{DK} = 0$$

$$F_{DK} = -8.175N = \underline{\underline{8.175N\ (compression)}}$$

LEARNING ACTIVITY #3

6) Can you determine the strength of the verticals and end posts? To determine the compressive strength of the verticals and end posts, we will use the strength vs. length graph we developed in Learning Activity #2. The verticals are 10mm x 6mm tubes, and each has a length of 12.5 centimeters. Thus the compressive strength of the vertical tubes is 43 newtons, as indicated below:

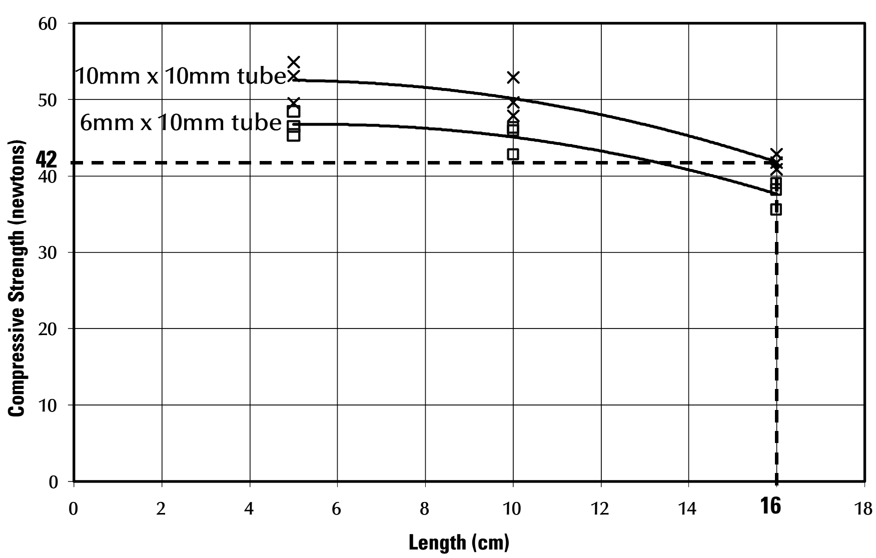


Determining the compressive strength of a 6mm x 10mm tube that is 12.5cm long

The end posts are 10mm x 10mm tubes. Since they are diagonal members, we must calculate their length using the Pythagorean Theorem:

$$L_{AI} = \sqrt{(10.0cm)^2 + (12.5cm)^2} = 16.0cm$$

As the graph below indicates, a 10mm x 10mm tube with a length of 16 centimeters has a compressive strength of 42 newtons.



Determining the compressive strength of a 10mm x 10mm tube that is 16cm long

7) **Can you calculate the factor of safety for all of the remaining truss members?** Once the internal member force and strength are known for each member in the truss, the corresponding factor of safety can be calculated using the equation

$$FS = \frac{Strength}{Internal\ Member\ Force}$$

The results are summarized in the table below.

Members	Force	Strength	FS
AB, FG	9.81 N (tension)	52	5.3
BC, EF	9.81 N (tension)	52	5.3
CD, DE	19.62 N (tension)	52	2.7
IJ, LM	19.62 N (compression)	50	2.5
JK, KL	22.89 N (compression)	50	2.2
AI, GM	15.70 N (compression)	42	2.7
BI, FM	0 N --	52	--
CI, EM	15.70 N (tension)	52	3.3
CJ, EL	12.26 N (compression)	43	3.5
DJ, DL	5.23 N (tension)	52	9.9
DK	8.175 N (compression)	43	5.3

8) **Why are some truss members stronger than they need to be?** For example, why did the structural engineer not use a doubled 3mm bar for Member CI, rather than the doubled 4mm bar she specified in the design? Making this change would not adversely affect the overall safety of the structure; yet reducing the member size would clearly use less material. And using less material ought to lower the cost of the structure, right?

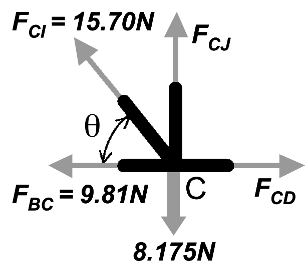
Member CI and a number of other members in the Grant Road Bridge are "too strong" because the structural engineer chose to use only a limited number of different member sizes in her design. In this case, she used only three—the two tubes and the doubled 4mm bar. She chose the 4mm bar so that the tension members with the largest internal force—CD and DE—would have a factor of safety greater than 2. Then she simply specified the same 4mm bar for all of the other tension members, knowing that this size would be more than adequate for members whose internal forces were lower.

To understand why the engineer chose to use a limited number of member sizes, just think about your own experience building the Grant Road Bridge. Suppose the engineer had designed every member with a safety factor of exactly 2. Most likely, each main truss would have required five different bar sizes and five different tube sizes. With so many different sizes, it would have taken you much longer to measure, cut out, and assemble the members. The connections would also have been much more complicated, and you would have been more likely to make construction errors—to put a 3mm bar where a 4mm bar is supposed to be used, for example. You would have saved some material, because every member would only be as strong at it absolutely needs to be. But this small reduction in material cost probably would not have been worth all of the extra work.

The same is true for the construction of a real structure—there can be substantial cost saving in using a few standard member sizes, because doing so can greatly simplify the fabrication and construction processes.

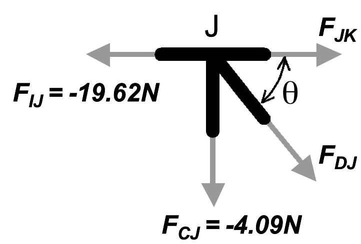
9) Can you apply the Method of Joints to analyze a truss? To analyze the truss with bottom-chord loading, follow exactly the same procedure as you did for the truss with top-chord loading. You will find that the calculation of reactions, and the analyses of Joints A, B, and I are *exactly* the same as for the top-chord loading. Starting at Joint C, however, the two solutions differ. The truss with bottom-chord loading has an 8.175N load at Joint C, as shown in the free body diagram. As a result, the x-direction equilibrium equation remains the same, but the y-direction equilibrium equation changes:

$$\sum F_x = 0$$
$$-F_{BC} - F_{CI}\cos\theta + F_{CD} = 0$$
$$-9.81 - (15.70)(0.6247) + F_{CD} = 0$$
$$F_{CD} = +19.62N = \underline{\underline{19.62N\ (tension)}}$$



$$\sum F_y = 0$$
$$F_{CI}\sin\theta + F_{CJ} - 8.175 = 0$$
$$(15.70)(0.7809) + F_{CJ} - 8.175 = 0$$
$$F_{CJ} = -4.09N = \underline{\underline{4.09N\ (compression)}}$$

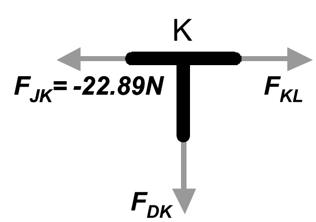
Next we analyze Joint J. In this case, the absence of an external load at the joint causes the y-direction equilibrium equation to change.



$$\sum F_y = 0$$
$$-F_{CJ} - F_{DJ}\sin\theta = 0$$
$$-(-4.09) - F_{DJ}(0.7809) = 0$$
$$F_{DJ}(0.7809) = 4.09\ \ F_{DJ} = +5.23N = \underline{\underline{5.23N\ (tension)}}$$

$$\sum F_x = 0$$
$$-F_{IJ} + F_{DJ}\cos\theta + F_{JK} = 0$$
$$-(-19.62) + (5.23)(0.6247) + F_{JK} = 0$$
$$F_{JK} = -22.89N = \underline{\underline{22.89N\ (compression)}}$$

Finally we analyze Joint K to determine F_{DK}.



$$\sum F_y = 0$$
$$-F_{DK} = 0$$
$$F_{DK} = \underline{\underline{0N}}$$

10) Can members with zero force be removed? Members BI and FM could be removed from our model of the Grant Road Bridge with no adverse consequences; however, they could not be safely removed from the actual highway bridge. In the actual bridge, Members BI and FM serve an important structural function—they support the floor beams attached to the truss at Joints B and F. These floor beams help to support the bridge deck and transmit vehicular loads from the deck to the main trusses. Thus, in the actual bridge, Members BI and FM are in tension and are essential to the structure's load-carrying ability.

Member DK could not be safely removed from either the model or the actual bridge. Without Member DK, we would have just one continuous member from Joint J to Joint L—there would be no reason for a joint at K. This new member—let's call it Member JL—would be twice as long as Member JK or Member KL. As a result, Member JL would be much weaker in compression than JK or KL. (Recall from Learning Activity #2 that compression strength decreases substantially with increasing length.) To keep Member JL from buckling, a considerably larger tube would be required. Thus, even though Member DK has no internal force, it effectively strengthens the structure by dividing Member JL into two shorter, stronger compression members.

It is also worth noting that, in an actual bridge, the internal force in Member DK would not be zero. It would actually carry part of the self-weight of Members JK and KL, resulting in a small compressive internal force. In our analysis, the internal force in Member DK is zero only because we assumed the self-weight to be zero.

11) Can you analyze a more complex truss? The Gallery of Structural Analysis Results (Appendix B) provides the calculated internal member forces for a wide variety of common truss configurations.