Module Overview	
For grade level(s)	Middle School: 6th & 7th Grade High School: 12th Grade
Duration	MESA Period: 2-3 sessions per week of 30-50 minutes each MESA Afterschool: 2 total sessions of ~60 minutes each MESA Saturday: 2 sessions for a total of 2 hours
Purpose	The purpose of this module is to better prepare students for the team math competition offered at MESA Day. In the Team Math Quest (TMQ) event, students will work together to solve math content covering multiple subject areas.
Objectives	Students will be able to: • Identify the math content areas measured by TMQ • State a general understanding of the CA Common Core Standards for Mathematical Practice • State the general rules of the Team Math Quest competition • Develop strategies for collaboration in a timed event
Standards Addressed	The content covered in Team Math Quest is very broad, both at MS and HS levels. Therefore, the Common Core standards addressed are extensive and cannot all be listed here. Instead, the Standards for Mathematical Practice are listed: 1. Make sense of problems and persevere in solving them. 2. Reason abstractly and quantitatively. 3. Construct viable arguments and critique the reasoning of others. 4. Model with mathematics. 5. Use appropriate tools strategically. 6. Attend to precision. 7. Look for and make use of structure. 8. Look for and express regularity in repeated reasoning.
Assessment	Students will be evaluated through practice exams and the official Team Math Quest competition at Preliminary and Regional MESA Days.
Additional Resources	Previous versions of actual Team Math Quest exams can be found on the CA MESA Intranet. For access, contact your MESA Director. http://mesa.ucop.edu/staff/mesa-day-rules/ Khan Academy: https://www.hkanacademy.org

Background

Over the years, Team Math Quest has changed in format several times, but the core of it has always remained the same. Group two or three students based on grade level and/or similar math course content and send them on a 'quest' to solve math problems from multiple disciplines. Unlike the Solo Math competition, students must work together to solve problems that are more challenging than a basic arithmetic equation and must do so in a timed format.

Basic Rules:

- Teams can be comprised of **2 OR 3** students as long as they meet the grade requirement.
- Only non-QWERTY calculators may be used. This is defined as calculators with standard numeric keypad configuration, versus a 'keyboard-like' device. Calculators may not be embedded into devices like an smartphone or tablet.

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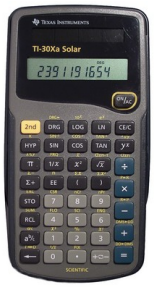

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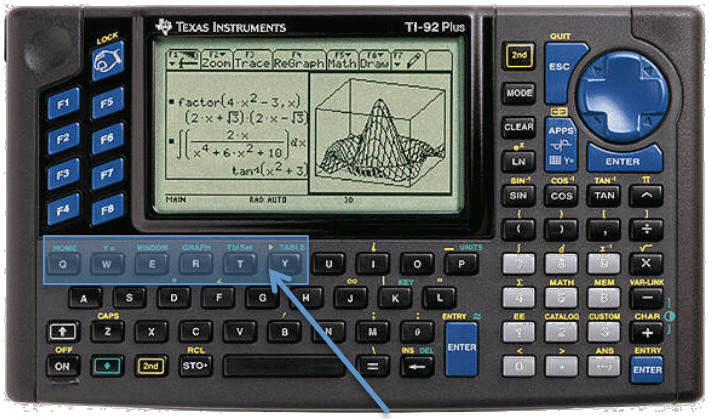


QWERTY= Keyboard

- See Rule #3 for full details. If you still have questions, contact your MESA Director.

Basic Rules (continued):

- Each exam is comprised of 30 problems. Each correct answer is worth **4 points**. Each incorrect answer is penalized **(-1) point**.
- Teams will have a total of **50 minutes** to complete the exam.

Changes for MESA Day 2016:

- Only grades 6, 7, and 12 are eligible to take Team Math Quest. The subjects covered in each exam are as follows:
  - For MS (6^{th}-7^{th}): General Math, Pre-Algebra, Algebra I
  - For HS (12^{th}): Alg I&II, Geometry, Trig, Pre-Calc/Math Analysis, Calculus
- All problems are multiple-choice, and the team must record their answers on the official Scantron provided by the Host Center.

Benefit To Society

Many students ask why study math. Others say "Who cares about math and when am I ever going to need it?"

Mathematician John Allen Paulos writes:

"As a mathematician, I'm often challenged to come up with compelling reasons to study mathematics. If the questioner is serious, I reply that there are three reasons or, more accurately, three broad classes of reasons to study mathematics. Only the first and most basic class is practical. It pertains to job skills and the needs of science and technology. The second concerns the understandings that are essential to an informed and effective citizenry. The last class of reasons involves considerations of curiosity, beauty, playfulness, perhaps even transcendence and wisdom." [Paulos, John Allen. *A Mathematician Reads the Newspaper,* p164-168. Basic Books (1995)]

Broadly speaking, studying math is critical because it is:

- Useful:
  - Mathematical problems abound in daily life
  - Mathematical proficiency is required for many jobs
  - Mathematics is essential for science, engineering, and research
- Important:
  - A mathematically informed citizenry will make better economic and political decisions about risk, policy, and resource allocation.
- Interesting:
  - "Mathematics, rightly viewed, possesses not only truth, but supreme beauty" (Bertrand Russell) and should be studied in its own right
  - The landmark accomplishments of mathematics stand alongside the masterworks of art and music as cultural triumphs that all educated persons should be able to appreciate

- Doing mathematics teaches patterns of problem solving and insight that transfer to other knowledge domains

Mathematics has been a part of a traditional western education since classical times -- the door to Plato's Academy in ancient Greece reportedly bore the inscription "Let No One Ignorant of Geometry Enter Here." In The Republic, Plato lays out a course of study for citizens of an ideal society. His higher education begins with Arithmetic, of which "all arts and sciences necessarily partake," and which "leads to the apprehension of truth:" [Plato. The Republic, Book VII (360 B.C.E) tr. Benjamin Jowett (1901)]

The Team Aspect

Working in teams is a much different experience than working individually. Whether the students like it or not, the reality is that working in teams is more reflective of the environment they will operate in more often than not. Collaboration is the name of the game now, so it's crucial that students understand what it's like to be a part of a team and how to achieve the group's goals in the most efficient manner possible.

Everyone has different strengths, and everyone can contribute something to a team's effort. Students have to be able to recognize each other's strengths and work together for the common good. There is also usually a deadline that teams must meet, and in the case of Team Math Quest, that is simulated by the 50 minutes allotted to answer 30 questions.

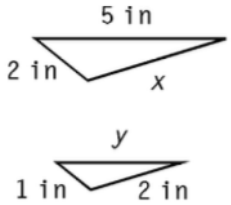
Lesson #1: Practicing Math

One of the aims of the new Common Core math standards is for students to not just be proficient with grade-specific content, but also to be fundamentally sound in the general approach to "doing" math. In general, the types of problems in Team Math Quest tend to be more complicated and require more thought than a simple arithmetic equation. Word problems are more likely and can require finding multiple sub-level solutions before getting the final solution.

Let's look at a simple problem and apply the MP standards:

Find the lengths of the missing sides in the pairs of similar figures.

1.



1. **Make sense of problems and persevere in solving them.**
   What type of problem is this? What information is the student being asked to find, and what information is already given? Of the information given, can any relationships be formed? What would be their first step in solving this problem? After arriving at a solution, do they question it or take for granted that it's the correct one? A common mistake is to leave out units, which would be inches in this case, or to have a partial solution, either for 'x' OR for 'y'. Students should always ask themselves: does my answer make sense and is it complete?
2. **Reason abstractly and quantitatively.**
   Perhaps the student is able to identify which sides of the triangles correspond to each other. If so, there was a specific path they took to arrive at that conclusion. Was that a 'reasonable' and logical path to take? Can they look at this problem in general terms without the specific numbers and units, and then apply the quantitative values of those general terms. In other words, how do similar triangles relate to each other, and once you know those relationships, can they apply them to specific triangle pairs such as these?
3. **Construct viable arguments and critique the reasoning of others.**
   In the classroom, the inability to practice this standard is most likely manifested in arguments between classmates. "My solution is right" or "you're method is wrong" are thrown out as justifications for their own methodology. As is often the case, multiple solutions and multiple methods can coexist. Can one student make a logical argument for his/her methods versus that of another? Perhaps even more importantly, can he/she formulate a valid critique of other methods? This practice, or lack thereof, can have significant consequences in a team math competition.

Are there multiple solutions to solving for x and y in this problem, or multiple methods for arriving at the same conclusion? Is one way 'better' than another and why?

4. **Model with mathematics.**
   This is deeper than just developing the correct proportion equations for finding the values of x and y. In this problem there is a visual representation of a problem, and students use a mathematical model to find its solution. Can they turn around and apply that specific model, or the reasoning behind, to other math problems or even real world scenarios. Consider a cooking scenario in which a recipe needs to be increased or decreased. How might this triangle problem be a model for determining the correct quantity of certain ingredients to achieve the desired servings?
5. **Use appropriate tools strategically.**
   In a setting like the TMQ event, aside from pencil and paper, another tool that is available to them is a calculator. Some students are constantly pushing buttons, regardless of complexity of the problem. Does this triangle problem require a tool like a calculator? Does it facilitate finding the solution, why or why not? There will be more complex problems in TMQ, are students adequately prepared to use a calculator or even basic hand arithmetic to find solutions?
6. **Attend to precision.**
   MS and HS students often overlook precision in mathematics. Whether it's the correct use of a symbol/sign, or indicating proper units, students must develop consistent and meticulous approach to doing math. This doesn't just mean precision in the final solution, but precision by using correct math terminology and definitions when discussing reasoning/methods with others. Historically this type of problem has produced many lost points because teams forget to include units in their final answer, or they only provide the solution for one variable. Attending to precision is attention to detail!
7. **Look for and make use of structure. &**
8. **Look for and express regularity in repeated reasoning.**
   The triangles in this problem may represent general structure or patterns in other types of problems. At first glance this looks like a geometry questions, but then students will see that it also has simple algebra. Can they take elements of this problem and apply it in other situations? Or maybe apply the reasoning/methodology in this solution to other problems? Practice makes perfect, and over time students will find it beneficial to master this practice, and certainly will pay dividends in a TMQ setting.

Lesson #2: Building Team Chemistry

The following two activities can be implemented to reinforce the team aspect of Team Math Quest. Feel free to modify or add your own team building exercises as you see fit.

Activity #1: Acid River/Acid Pond

MATERIALS NEEDED:

- 1 long rope between 10-15 feets
- At least 2 different lengths of rope (i.e. 4ft and 6ft)
- 2 Small plastic buckets with or without handle
- 3-4 balls (i.e. tennis ball, foam golf ball, etc.)
- Stopwatch
- ***This is per group of 3-5 students. More materials will be needed to do this activity simultaneously with multiple groups.*

GOAL:
Transfer all the balls from one bucket to the other using only the materials provided

SETUP:
Form a circle on the ground using the 1 long rope and place one of the buckets in the center of the circle. Place the other bucket anywhere else outside the circle and put the balls inside the bucket. Give the team the remaining varying ropes. Teams can be 3-5 students.

RULES:

1. Your team is free to use any and all of the materials provided for you (small ropes, long ropes, etc.)
2. No one can stand inside the circle OR put any body parts inside the circle. Imagine the circle is projected upwards from the ground to the ceiling. This imaginary cylinder is off limits. If someone's arm/hand/foot goes inside this area, they will lose the ability to use that arm/hand/foot for the rest of the activity.
3. You will be timed.

REFLECTION:

1. How did the team approach the task?
2. Did you create a workable plan and was it implemented?
3. Were you a leader or a follower?
4. For those of you that lost use of a foot/hand/arm during the activity, how did you handle this?
5. If you were to do this activity again, what would you do differently?
6. What did you learn from this activity and how can you translate it to the TMQ event?

Activity #2: Making the Shape

MATERIALS NEEDED:

- 1 long rope, at least 15 ft, per group

GOAL:
Work together to create the given shape under given conditions.

SETUP:
Divided students into teams of 4-5 and give each team one long rope. If working with multiple groups, have groups spread so they have some room to expand.

RULES:

1. This exercise requires you to have our eyes closed at all times
2. You must have your hands on the ropes at all times, never letting go
3. Be careful not to slide your hands around the rope as to not have any rope burn
4. I will give you a shape (e.g. square) and you must form it as a group
5. Lift the rope with your hands and keep your eyes closed

Script for shapes:

- Form an Isosceles Triangle
  - Whenever ALL of you think that you have the shape—there must be a consensus—slowly put your shape down on the ground, let go and open your eyes.
  - How did you feel?
- Form a Parallelogram
- Form a Hexagon
  - How did you feel?
  - Did anyone open his or her eyes?
  - Please put your hands in the middle as a fist (show them as an example)… close your eyes… please put your thumb up if you opened your eyes during the exercise… please put your thumb down if you did not open your eyes during the exercise.
  - Please put your hands down… open your eyes.
  - Tell them how many people opened their eyes (if no one opened their eyes, tell them if you, as a facilitator, saw anyone open their eyes during the exercise.

***Feel free to add your own shapes and create your own restrictions, such as "you can open your eyes but you cannot talk".*

REFLECTION:

1. Were you a leader or a follower?
2. What do you think was the purpose of this exercise?
   - The purpose of this is to show them how to work through different processes without all of your resources (sight).

- Communication
- Team work
- The importance of listening and giving everyone a voice

3. How will this help you in a setting like Team Math Quest?

Practice Exams

SPECIAL NOTE: The practice exams offered in this guide are not in multiple-choice format. They are actual exams used in previous years using the free response format. Students should understand that they will be providing their final answer on a Scantron, rather than a blank score sheet.

The following exams and problem sets can be used as preparation for this year's TMQ exam:

For Middle School (6th and 7th Grades):

- 2009 Junior Preliminary Category I: Problems 1-30
- 2009 Junior Preliminary Category II: Problems 1-15
- 2009 Junior Regional Category I: Problems 1-30
- 2009 Junior Regional Category II: Problems 1-15
- 2013 Junior Level Exam: All problems **except #4, 7, 10, 13, 20, 23, 26 (these are Geometry-based)**
  (This is also the version that most closely resembles what the 2016 exam will look like)

For High School (12th Grade):

- 2009 Senior Preliminary Category A, B, & C: Problems 1-30
- 2013 Senior Level Exam: Problems 1-30 *(this version most closely resembles what the 2016 exam will look like)*